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Repricing Made-To-Order Production Programs.

Mehmet Murat Tarimcilar
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Repricing made-to-order production programs

Tarimcilar, Mehmet Murat, Ph.D.

The Louisiana State University and Agricultural and Mechanical Col., 1987

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REPRICING MADE-TO-ORDER PRODUCTION PROGRAMS

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and Agricultural
and Mechanical College in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy
in
Interdepartmental Program in Business Administration
(Quantitative Business Analysis)

by
M. Murat Tarimcilar
B.S., Bogazici University, TURKEY, (1980)
M.S., Louisiana State University, (1987)
August 1987

ACKNOWLEDGEMENTS

I owe a debt that I am not sure can be repaid to my major professor, Thomas R. Gulledge, Jr. I've been inspired by his rigor and enthusiasm in teaching, research, and most recently in dissertation advising. I'm grateful to have had the opportunity to work with him.

I would like to thank my dissertation committee for their cooperative effort and help.

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ABSTRACT

Planned procurement quantities in made-to-order production programs are often altered after production has started. Cost analysts are concerned about providing cost estimates for alternatives to ongoing programs.

This research develops a new model for repricing made-to-order production programs. It is an effort to integrate a theoretically developed production phase prediction model into a comprehensive repricing model. The production model is defined by a cost function that employs the impact of two cost determinants, production rate and learning. It is supported by economic theory and is consistent with our knowledge of made-to-order production process. The theoretical model links the direct costs to delivery schedules under the assumption that the firm attempts to optimize production rate over time. Actual delivery schedule data are used in the estimation of the direct variable cost. The repricing model, besides the direct variable cost, also considers the effects of fixed cost, business base, and expenditure profile.

An explicit decision support system is developed that utilizes the model. This support system is used to test the validity of the model and to illustrate the estimation results.

CHAPTER I

INTRODUCTION

Planned annual procurement quantities for defense weapon systems are often altered after production has started. In many cases, these alterations result in changes in the quantities of weapon systems to be acquired in future years. As part of the review of these changes, cost analysts generate cost estimates for alternatives to ongoing and planned programs.

It is imperative that new techniques be developed and old techniques be improved to obtain better cost estimates for weapon systems procurement programs. With these new techniques, a better understanding of model determinants is also required. The cost impacts of policy decisions must be available if these decisions have the desired results in the dynamic world of systems acquisition.

The importance of this problem is motivated by the fact that Department of Defense planners must provide procurement strategies for many weapon systems. These strategies include estimates of the unit price of each system for many hypothetical procurement quantities. These estimates are provided several times each year, and therefore it is impossible to perform an extensive industrial engineering cost analysis for each scenario. Also, for proprietary and accuracy reasons, it is unrealistic to request that the contractor provide these cost estimates. This important cost analysis problem is called the made-to-order repricing problem.

This research will focus on repricing an aircraft program by considering changes in quantities to be placed under contract in future years and by estimating the associated cost without consulting the producer. Due to limited access to the contractor's accounting records, difficult demands are placed on Department of Defense planners. They must reprice many alternative procurement scenarios while having limited access to detailed contractor cost data.

There has been considerable research related to the repricing of annual contracts. Most of these studies use learning curve methods that treat all costs as being variable. However, over the years it has been observed that there has been a shift in the composition of costs from the direct to the indirect categories. This shift has degraded the performance of traditional cost estimating methods that treat all costs as being variable or varying directly with the quantity produced.

Balut, Gulledge and Womer (1986) present an approach to the aircraft repricing problem. This approach explicitly considers the effect of quantity invariant costs on total procurement price. The model combines recent developments in the operations research literature with more traditional cost accounting techniques to assign a total price to each procurement lot. At the heart of this model is a procedure for separating costs into fixed and variable categories. That is, cost accounting data are aggregated into two categories: direct and indirect costs. The indirect category contains some items that are fixed and some items that vary with quantity produced. The Balut, et. al. (1986) model requires that the fixed and variable parts of overhead costs be separated. This requirement is presented in Figure 1.1.

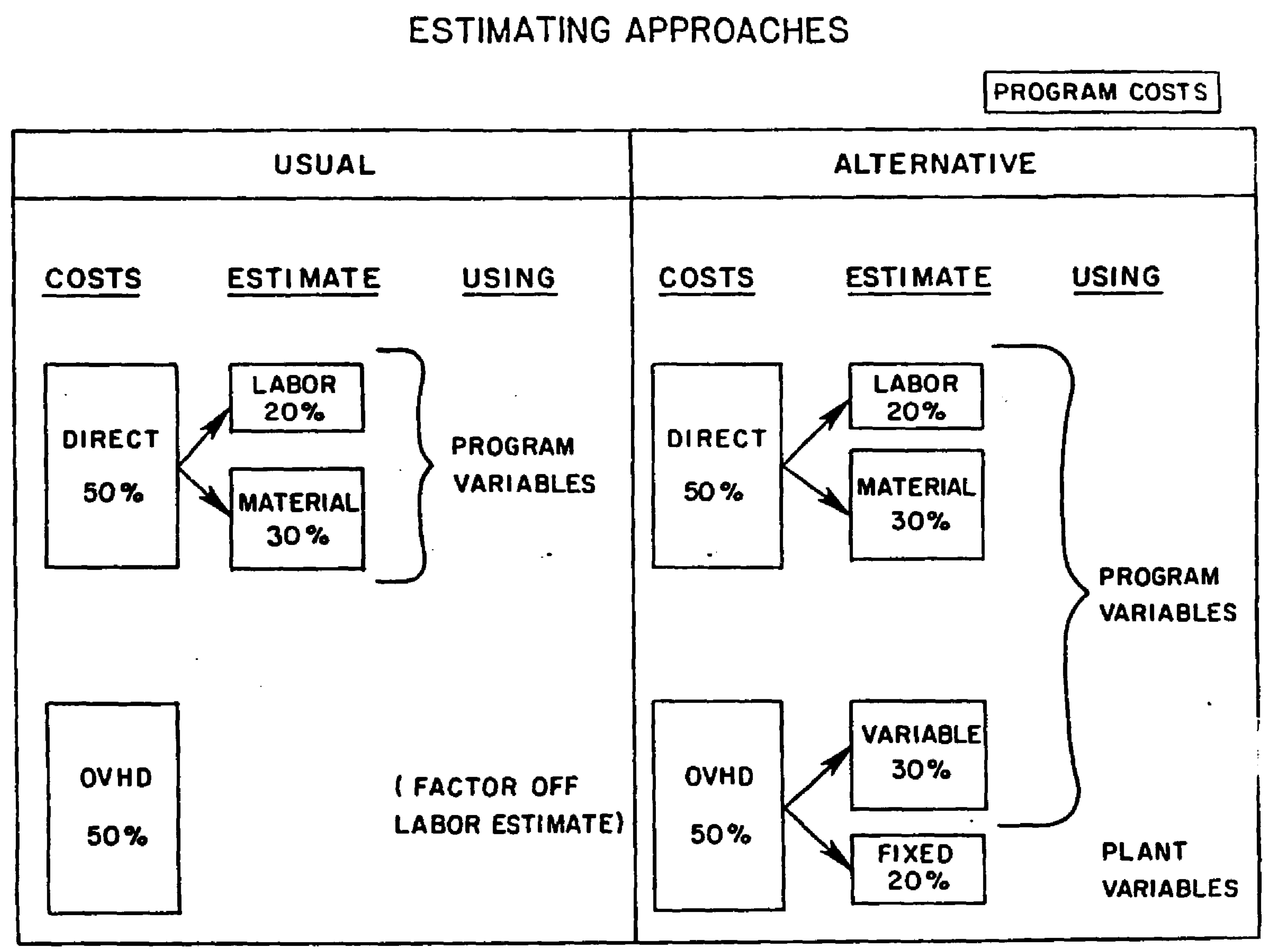


Figure 1.1 Alternative Estimating Approaches.

In the left panel of Figure 1.1, the traditional cost accounting approach to separating costs is presented. In the right panel, the segregation required by the repricing model is presented. In short, plant-wide costs must be categorized as fixed and variable, not direct and indirect.

After the separation, a statement of the repricing problem is deceivingly simple. Program variable costs are modeled with the learning curve. The fixed costs are then allocated across all units in the contractor's plant. Of course, as will be seen in a later chapter, this is a very simplistic statement of the problem.

Over the years, it has been demonstrated repeatedly [see, for example, Gulledge and Womer (1986)] that the learning curve is not appropriate for modeling variable cost unless production rate is constant. The effects of changes in production rate are not taken into consideration by methods which employ only learning curve techniques (Womer, 1979). The theoretical foundations for investigating production rate impacts on costs have been considered by economists for many years. On the other hand, many engineering studies consider cumulative output to be the most important cost determinant. Alchian (1963) implicitly combined progress functions with economic theory in a study related to military airframes. Since then, a plethora of research has occurred in this area. These efforts will be discussed in detail in the next chapter.

This dissertation will focus on introducing a production rate factor into a model for repricing aircraft procurement programs. As previously mentioned, the current successful repricing model explains variable cost with a learning curve. Also, as previously mentioned, this is not correct unless production rate is constant, a rare case in aircraft production. This research provides a methodology for incorporating production rate effects into a repricing model, and the resulting predictions are compared with those from the Balut, et al. (1986) model.

CHAPTER II

LITERATURE REVIEW

Historically, economists have used neoclassical economic theory to specify the functional form of the cost function. This function relates cost to output rate and prices while ignoring other cost determinants. Beginning with Wright's (1936) seminal work, cost was modeled as a function of cumulative output, while ignoring other cost determinants. Wright's approach is a popular industrial engineering method for analyzing cost. Prior to the late 1960's, no serious attempts were made to integrate the approaches. While a number of researchers were aware of the fact that both output rate and cumulative output are significant determinants of cost, an applicable link between economic cost theory and engineering learning curves was not achieved.

This literature review spans more than a quarter of a century. Alchian (1959) introduced the integration idea; however, it was not until Washburn's (1972) work that the first applicable integrated model appeared. The scope of most early studies was restricted to "made-to-order" production, mainly government contracted airframe production programs. Womer and Gulledge (1983) developed a model for airframes which considered the cost impacts of learning, production rate, and facility size on total program costs. They were the first researchers to provide an applicable a priori specified model that integrated the learning curve with neoclassical economic theory.

Most of the early cost models examined only one component of cost, direct labor requirements. It was much later [Balut, et al., (1986)], before a successful model was presented for analyzing total aircraft cost. Nonetheless, the early studies provide the groundwork for the research of this dissertation, so they are included in the literature review.

Alchian, A. A. (1959)

Conjectures about the relationships among cost and outputs were presented as propositions in a paper by Alchian (1959). Cost is defined as change in equity, the capital value concept of cost. Alchian states that numerous factors can affect production cost, but he concentrates only on three factors in this paper:

1. output rate, $x(t)$,
2. total planned volume, V, and
3. program delivery dates.

These three characteristics are related by the following relationship:

$$V = \sum_{T}^{T+m} x(t), \tag{2.1}$$

where T is the time for the first delivery and m is the length of the interval over which the output is made available. Note that one of these variables is constrained while the remaining three are independent, so cost is presented as $C=f(V,x,T,m)$.

Alchian states several propositions that define how changes in several variables impact cost. The first proposition is

$$\left.\frac{\partial C}{\partial x}\right|_{\substack{T = T_0 \\ V = V_0}} > 0. \tag{2.2}$$

This proposition states that as output rate increases while total volume remains constant, cost increases. Or, the faster the rate at which a given volume of output is produced, the higher its cost.

Proposition 2 is

$$\left.\frac{\partial^2 C}{\partial x^2}\right|_{\substack{T = T_0 \\ V = V_0}} > 0. \tag{2.3}$$

This proposition states that the increment in cost is an increasing function of output rate.

The third proposition relates to the cost impact of planned volume, i.e.,

$$\left.\frac{\partial C}{\partial V}\right|_{\substack{x = x_0 \\ T = T_0}} > 0. \tag{2.4}$$

Cost increases with volume. Since x is constant, m must become an adjustment variable and hence the production time interval becomes longer. More resources are required to produce more output, and therefore cost must rise. Furthermore,

$$\left.\frac{\partial^2 C}{\partial V^2}\right|_{\substack{x = x_0 \\ T = T_0}} < 0. \tag{2.5}$$

If planned output is increased uniformly, cost will increase by diminishing increments. The fifth proposition relates to average cost, i.e.,

$$\left.\frac{\partial^2 C/V}{\partial V}\right|_{T = T_0} < 0. \tag{2.6}$$

Since marginal cost is falling, average cost also declines. Proposition six states that

$$\left.\frac{\partial^2 C}{\partial V \partial x}\right|_{T = T_0} < 0. \tag{2.7}$$

Marginal cost with respect to increasing output rate decreases as total planned volume increases. Proposition seven is concerned with the production time horizon,

$$\left.\frac{\partial C}{\partial T}\right|_{\substack{x = x_0 \\ V = V_0}} < 0. \tag{2.8}$$

The longer the time between the decision to produce and the delivery of output, the less the cost.

The last two propositions are verbal. The eighth proposition addresses short- and long-run effects on cost. In the short run at least one input is fixed, whereas in the long run, all inputs are variable. Even though the distinction between these two costs helps explain the paths of price and output over time as demand varies, Alchian states that for any given output program, only one type of cost can be considered: the cheapest cost. Total, average, and marginal costs decrease as T increases, but with different rates.

The ninth proposition relates to learning. As the total quantity of units produced increases, the cost of future output declines; knowledge is increasing as a result of production. The implication is that cost will be lower in the future.

The difference between propositions four and nine should be noted. In both cases, cost changes as a function of planned output, but in proposition four the change is due to changes in technique. In proposition nine, since planned output is larger, accumulated experience and knowledge will be higher. This is because knowledge is proportional to accumulated output. In the industrial engineering literature, this ninth proposition is known as the learning curve effect.

Two of the features emphasized in this paper are the distinction between output rate and planned volume, and changes in technology that are different from changes in technique. These two features suggest that cost is lower for larger quantities of a product because of cumulative output. This feature emphasizes the importance of the variable, V, as a determinant of production cost.

Conway, R. W. and Schultz, A. Jr. (1959)

Progress in production effectiveness is a function of the time horizon of the manufacturing process. Conway and Schultz argue that this progress is not necessarily due to learning which they define as improvement in performance at a fixed task. It may be the progress of an organization which learns to do its job better by changing the tasks of individuals. Therefore, this paper relates to progress functions as opposed to learning curves.

Conway and Schultz examine Wright's (1936) unit learning curve model. This model suggests that

$$y_i = ai^{-b}, \tag{2.9}$$

where i = production count,

y_i = labor hours required for the i^{th} unit,

a = labor hours required for the first unit, and

b = a measure of the rate of reduction (progress).

The authors try to determine if the cumulative average model, which is given as $\tilde{y}_i = y_i i^{-b}$ (where $\tilde{y}_i$ is the average hour per-unit taken over all units from the first to the i^{th}.), is a more efficient model than the unit curve model. However, both theoretically and empirically, they could not find any sufficient superiority of one alternative over the other. They do note that the cumulative average formulation smoothes the data, so instead of using only one model, they decided that the two models are complementary and should be used together for a better analysis.

The authors also analyze the sum of two models:

$$y_{1i} = y_{11}\ i^{-b_1}, \tag{2.10}$$

$$y_{2i} = y_{21}\ i^{-b_2}, \tag{2.11}$$

and conclude that "if the model is assumed to hold for two separate production processes, it cannot also be assumed to hold for their sum unless the separate curves have the same slope which will not in general be the case."

The above suggests an estimating technique for different classes on the production line. The idea is to classify total cost into categories of similar progress characteristics. Progress functions are estimated separately for each category, and then the progress functions are summed. Even though the slopes are different for each class, the curves show the same characteristics for different classes of the same production process, i.e., leveling off at the end for the assembly of large units. The authors conclude that the procedure looks promising.

The estimating procedure is based on estimates of the labor hours required to produce the product. When the labor hours are divided into meaningful categories, labor content should have reasonably uniform behavior with respect to the reduction rate. Once the labor requirements are obtained, overall cost may be projected by using average hourly wage rates, overhead rates as a proportion of labor cost, etc. The estimates for each category will then be aggregated either graphically or by using tables of progress function factors.

Conway and Schultz do not integrate production rate effects into their model. They conclude that there is progress in production, but all of the progress is attributed to experience over time. One additional noteworthy conclusion is that there are significant differences in patterns of progress for different industries and different firms. The authors also suggest the use of forecasting methods for estimation, but they do not present any empirical results.

Hirshleifer, J. (1962)

Hirshleifer reviews and attempts to justify Alchian's (1959) propositions. He compares Alchian's reformulation of the cost function with classical economic cost functions. Instead of treating the two theories as contradictory, he shows that the classical shape of the marginal cost curve is consistent with Alchian's propositions. To reach this conclusion, Hirshleifer assumes a fixed length of production and that planned volume moves proportionately with output rate. He also argues against Alchain's approach to the classical definition of the long- and short-run cost concept. The discussion mainly relates to the significance of volume since both classical economic theory and Alchian agree on the importance of output rate.

After classifying firms into different groupings, Hirshleifer decides which groups are appropriate for Alchian's model and which are more appropriate for the neoclassical model. He concludes that firms which produce to order can usually be modeled using Alchian's methodology. The significance of the effect of total volume on cost is certain for such firms; for example, a firm producing military aircraft to government order.

Hirshleifer agrees that decreases in cost due to changes in planned volume (learning) may occur, but he argues against Alchian's extension, which is, even if knowledge is constant, the marginal cost with respect to V will still be declining. Hirshleifer says this can happen only under the assumption of perfect knowledge about future production, which is not very realistic. He also gives some examples for the case where cost increases as volume rises. This behavior is usually associated with production processes that are labor intensive. The reason for the cost increases is the biological phenemenon of fatigue. Hirshleifer also examines firms that produce to aggregated rather than individual orders to see if they may be modeled by Alchian's model.

Finally, Hirshleifer examines firms fitting the classical model, given that V is not infinite and there is uncertainty about future production. In this case, x and V will be stochastic; however, for simplicity, the author assumes that x is proportional to V. He shows that marginal cost rises as x and V increase proportionately and concludes that with a rising marginal cost curve, the U-shaped average cost curve can be explained as a special case of Alchain's model. Hirshleifer shows these results with two additional propositions:

1. $$\left.\frac{\partial C(\alpha,x)}{\partial x}\right|_{V = \alpha x} > 0, \tag{2.12}$$

2. $$\left.\frac{\partial^2(\alpha,x)}{\partial x^2}\right|_{V = \alpha x} > 0, \tag{2.13}$$

where α is the proportionality constant between V and x.

Therefore, Hirshleifer's primary assertion is that Alchian's model is not an alternative to the classical model, but an extension that provides a detailed and better fit for some cases. Alchian's model mainly states that scheduled production volume has a different effect on cost from the effect of output rate. Hirshleifer concludes that Alchian's propositions are useful in explaining situations that are difficult to model by classical economic theory.

Preston, L. E. and Keachie, E. C. (1964)

Preston and Keachie try to integrate the economic and the industrial engineering approaches for examining costs and outputs. They present both graphical and statistical analyses to support their hypothesized relations. The authors examine the U-shaped on L-shaped cost functions of economic theory and hyperbolic learning curves from the engineering literature (i.e., static cost functions and progress functions).

In their model, cost and progress functions are integrated by considering three variables:

C_t = production cost per time period,

q_t = output per production period (lot size),

V = the accumulated level of total output.

The authors use regression models to examine the "cost-output" relation. Although the estimation could not detect the rising phase of the short-run cost curve, in several of the models the parameter estimates are still significant at the 5% level.

Unit total cost and unit labor cost are used interchangeably as dependent variables in the models; lot size and cumulative quantity are both included as independent variables. The results show that the relative importance of lot size as an explanatory variable is greater for unit total cost than for unit labor costs, and the relative importance of cumulative output is correspondingly less when the dependent variable is total cost. It is also hypothesized that the decline in unit costs attending the accumulation of output over time is well described as the learning phenomenon. Finally, it is noted that from a statistical point of view, accumulation of output experience is more important when considering labor cost than total cost.

Preston and Keachie estimate their cost function empirically without using the theoretical support of economic production theory. Also, ordinary least squares may not give good estimates because of the collinearity between cumulative output and output rate (Camm, et al., 1987a). The authors conclude that both cumulative output and output rate are significant variables when explaining production cost.

Oi, W. Y. (1967)

In neoclassical economic theory, output growth is explained by increases in annual flows of labor and capital inputs. Empirical results show that these increases alone are not enough to explain economic growth. Oi claims that even though learning by doing, which is the main idea of progress functions, is important, it is not an endogenous part of growth.

If neoclassical production theory is extended, Oi argues that important features of the progress function can be derived. Since the progress function is a dynamic concept, it does not fit within the static analyses of neoclassical economic theory. Starting with the assumptions in Hicks' dynamic model, Oi states two important theorems about the behavior of cost.

> Theorem 1: The cost of producing any given flow of output can be reduced by postponing the period of delivery.

This implies that a firm can achieve intertemporal factor substitutions to minimize cost, which Oi claims are precluded by neoclassical cost theory.

> Theorem 2: The cost of an integrated output program in which the plan is to produce output flows in several consecutive periods will be lower than the combined cost of unrelated output programs that yield some vector of dated output flow.

This theorem implies that the firm can reduce cost by making production plans in advance because of the complementarities of joint production. Oi states that output changes are often driven by neoclassical theory's "economies of joint production" concept. However, he says, all those changes are attributed to the phenemonen of learning where, in fact, at least part of those are a result of the economies of integrated output programs.

Oi also reviews Alchian's (1959) propositions and concludes that all these propositions are logical consequences of his modified dynamic theory of production. In the conclusion, the author states that his dynamic production theory, along the lines of Hicks, provides an essentially neoclassical explanation for the progress function. The gains in production can be explained by time-dependent production plans which are implied in neoclassical theory, not necessarily by technical change of learning.

Oi's work is mostly a verbal exposition. He does not specify a functional form to be used in applications, but his research is important because it considers the importance of production theory in the derivation of cost functions.

Rosen, S. (1972)

This paper investigates a model for a firm whose production technology is affected by knowledge. Rosen takes knowledge as an input and learning as an output from the production process.

The author considers two cases when defining knowledge:

1. Knowledge is vested in the owners or managers. Knowledge may be identified with pure "entrepreneurship", having to do with the ability to organize and maintain complex production process. Here the asset is not salable, though owners may rent the services of their knowledge elsewhere. Therefore, the market value of the firm (apart from its physical capital) in the absence of a tie-in contact with current owners is zero.

2. Knowledge is vested in the firm. Knowledge gained by the firm can be used in the absence of entrepreneurs, so it is transferable.

The most important difference between the two cases is the length of the time horizon. In the first case the horizon is finite, depending on the life time of the owners; in the second case, it is infinite since knowledge is transferable.

Rosen builds a model with the assumption of a finite time horizon on the optimum accumulation of knowledge. The variables used in the model are:

x_t = the amount produced in period t,

L_t = composite market input use rate in period t,

Z_t = accumulated knowledge related to production at the beginning of period t.

The constraints for the dynamic model are

$$x_t = F(L_t, Z_t), \tag{2.13}$$

and

$$\Delta Z_t = Z_{t+1} - Z_t = \beta x_t \tag{2.14}$$

where β is a constant.

Since the second constraint can be written as

$$Z_t = Z_0 + \beta \sum_{j=0}^{t-1} x_j, \tag{2.15}$$

and x_t is a function of Z_t, equation (2.15) is a progress function where knowledge is indexed by accumulated output.

Assuming that p is the market price for output, and w is the price of the composite input, the objective function in the dynamic model with discounting is

$$V_N(Z_0) = \max_{L_0} \{(px_0 - wL_0) + V_{N-1}(Z_1)/(1+r)\} \tag{2.16}$$

where V_N is a function of knowledge at the beginning of the time horizon.

The proposed solution requires solving the problem for each period by considering what happens in future periods. This logic is the same as the logic behind dynamic programming. By using this method, Rosen finds the maximum present value at the beginning of any period as a function of initial knowledge in that period.

Rosen specifies an alternative formulation by changing the second constraint to $\Delta Z = (1/\gamma)L_t$ where γ is a constant. This formulation implies that learning is proportional to the input rather than the output.

The functional equation in this model becomes

$$V_N(Z_0) = \max_{Z_1} \{pF[\gamma(Z_1 - Z_0), Z_0] - w\gamma(Z_1 - Z_0) + V_{N-1}(Z_1)/(1+r)\}. \tag{2.17}$$

The procedure for obtaining the n stage solution is the same as above. Present value is maximized in each time period throughout the planning horizon.

After analyzing the model, Rosen states that both the rate of investment and the final stock of knowledge in each period increase as the degree of diminishing returns to input L and stock Z decrease. This implies that the shorter the horizon, the less knowledge accumulated.

Rosen's model is an answer to Oi's claim that knowledge is an exogenous factor in diminishing cost. Rosen states that the neglect of accumulated knowledge in cost and production studies causes some researchers to attribute the effects of exogenous technical change to increasing returns to scale in inputs. He further suggests that the firm may find it profitable to incur costs in connection with learning in order to substitute knowledge for the purchase of future inputs.

This research is important because it was the first work that used dynamic programming theory as a theoretical foundation. Rosen's functional form was the basis for further research on learning augmented planning models [Gulledge, et al., (1985) and Womer et al., (1986)].

Washburn, A. R. (1972)

Washburn's (1972) model is related to aircraft production, but it is applicable to any production process which satisfies the following postulates:

1. the market is modeled as a constraint on total quantity produced instead of production rate,
2. profits are discounted, and
3. the product is produced on an assembly line and cost decreases throughout the production period.

Washburn develops a continuous model of a learning augmented production process. Both cumulative production and production rate are included in the model. The author defines $N(t)$ as a total production up to time t, and therefore $\dot{N}(t) = dN(t)/dt$ is the production rate. When cash flow is discounted, the problem of maximizing profit becomes a calculus of variations problem of the form

$$\max \int_0^t F[N(t),\dot{N}(t)]\, e^{-\alpha t}\, dt \tag{2.18}$$

s.t.

$$N(0) = 0,$$
$$N(T) = V, \tag{2.19}$$
$$\dot{N}(T) \geq 0,$$

where α is the discount rate.

In this model the production facility is assumed fixed, so the production rate can only be increased by using more labor. This requires the use of overtime or hiring additional manpower.

Washburn uses the notation C to define the standard crew and SMH to define a standard man hour. A standard man hour is the amount of work accomplished by one man in one hour when he is one member of a standard crew. This implies that efficiencies remain constant as long as the crew size is less than standard, and diminishes as the crew size becomes larger.

Next, Washburn defines the following proportionality relationship:

$$\text{Cost/SMH} = wh(\dot{x}/C) \tag{2.20}$$

where $h(y)=1+ay^b$ with $a,b > 0$, and $h(0) = 1$;

$\dot{x}$ = the rate of production of SMH; and

w = the basic wage.

Further, Washburn assumes that the amount of work required to pass the Nth unit through the ith position is given by $H_i g(N)$, where $g(N)$ is an improvement curve. With these definitions, the total money spent on labor is stated as

$$L(N,\dot{N}) = w\ \Sigma\ \dot{N}H_i g(N)h\ [\dot{N}H_i g(N)\ /\ C_i]. \tag{2.21}$$

Washburn solves the model by using calculus of variations to obtain the relation

$$\ddot{N} = \alpha \frac{g(N)f'(\dot{y}) - P\ /\ wH}{g(N)f''(\dot{y})[Hg(N)/C]} - \frac{(\dot{N})^2 g'(N)}{g(N)} \tag{2.22}$$

where $\dot{y} = \dot{N}\ Hg(N)/C$ is called muscle factor, and $f(\dot{y})=\dot{y}h(\dot{y})$.

This solution leads to the following theorem:

> If $\dot{N}(t)$ is optimal for this model, and if $\dot{N}(H)=0$ for $t_1<t<t_2$; then either $t_1 = 0$ or $t_2 = T$, which implies that there are no internal gaps in the production.

The results are applied to three different markets:

1) fixed,

2) time limited, and

3) quantity limited.

In a fixed market, n units should be produced in a fixed time, T, i.e., $N(0) = 0$, and $N(T) = n$. In this market, optimal production will be zero over an initial or terminal interval depending on whether total profit is negative or positive.

In the time-limited market, the products are sold as fast as they are produced up to time T. In this market, the learning effect dominates production inefficiency, and an infinite production rate will lead to infinite profits.

For a general market type, the production problem becomes

$$\max \int_0^T [F(\dot{N},N) - \lambda(t)]\ e^{-\alpha t}dt + \{P[N(t),T]\ e^{-\alpha t}\} \tag{2.23}$$

where $\lambda(t)$ is overhead expenses, and $P(n,t)$ is the maximum total profit that can be made from t onwards if total production at t is n. Note that $P(n,t)=0$ in both time- and quantity-limited markets. Washburn illustrates the determination of optimal production rate by using an airplane production example.

The main point of this model is that a particular objective function with several price parameters is general enough for many production problems given the stated assumptions. Washburn was the first researcher to establish a functional form and generate an optimum production schedule when delivery time and planned quantity are specified in advance. It was the first applicable continuous model that integrated learning with production rate in production functions.

Spence, A. M. (1982)

Spence developed a model of competitive interaction in an industry where unit costs decline with accumulated output. A learning curve which relates unit costs to accumulated volume is included in the model. In such a situation, Spence defines short-run output as a type of investment since the learning at that stage helps to reduce future cost. This means that along the firm's optimal output path, marginal short-run profits as a function of output are negative.

Keeping all those relations in mind, Spence builds his model for a single firm with the following variables:

$x(t)$ = output of the firm at time t,

$R[x(t),t]$ = revenue of the firm at time t,

$y(t) = \int_0^t x(\tau)d\tau$, the accumulated output at time t, and $\Theta(y)$ is the general functional form for a learning curve. This curve, $\Theta(y)$, is a declining function of y with $\Theta(0) = 1$. Also, $\Theta(y)$ approaches zero as y approaches zero and $\Theta(y)x$ is the cost related to learning.

The firm's profits at time t are revenue less cost, i.e.,

$$\pi(t) = R(x,t) - mx - C_0\Theta(y)x, \tag{2.24}$$

where m and C_0 are model related constants. Thus the objective function becomes

$$\max V = \int_0^T \pi(t)\ dt. \tag{2.25}$$

Developing this model, Spence defines $\Gamma(y) = \int_0^y \Theta(v)dv$, and $d\Gamma/dt = \Theta(y)x$, where $\Gamma(y)$ is an increasing concave function that represents the area under the learning curve. This implies that total costs are

$$\int_0^T C_0\dot{\Gamma}dt = C_0\{\Gamma[y(T)] - \Gamma[y(0)]\}. \tag{2.26}$$

Since $y(0) = 0$ and $\Gamma(0) = 0$, total costs are $C_0\Gamma[y(T)]$, which means the total costs for a firm are equal to the area under the learning curve between 0 and T.

With this definition, the objective function can be written as

$$\int_0^T R(x,t) - mx - C_0\Gamma[y(T)]. \tag{2.27}$$

By using the calculus of variations the optimal path can be found.

Spence extends his model to include the multi-firm problem and competition. He introduces an open-loop equilibrium concept in which each firm selects the optimal output path given the paths of competitors. The assumption is that all paths are optimal for each firm.

Here $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]$ and the revenues are $s_i(x,t)$ for firm i. The analysis is the same as in the single-firm problem. At an optimum

$$\frac{\partial s_i}{\partial x_i} = C_0 \, \Theta[y_i(T)], \qquad t \in [t_i, T], \tag{2.28}$$

where $y_i(t) = \int_{t_i}^{t} x_i(\tau) \, d\tau$.

Spence calculates a dynamic equilibrium for different values of the parameters in the model. The demand curve is given by $x = b_0 e^{\delta t} p^{-\beta}$ where β is an elasticity value, δ is the growth rate of demand, and b_0 is a constant. The learning curve given by

$$\Theta(y) = M_0 + C_0 e^{-\lambda y}, \tag{2.29}$$

where λ determines the rate at which unit cost declines with volume or the speed of learning, M_0 is the minimum possible unit cost, and C_0 is the cost disadvantage of a starting firm.

In the calculations, Spence holds some parameters constant and solves the model. He presents numerical results, and generally the conclusions may be stated as:

1. The learning curve can create substantial barriers to entry, similar in effect to ordinary economies of scale in the static sense.
2. Moderate learning rates create the greatest barrier to entry. Very rapid learning, in particular, does not create structural or performance problems.
3. With moderately rapid learning and a reasonable range of time horizons, entry ceases; typically with three or four firms.

4. Market performance improves sharply from monopoly to two and three firms and rather slowly with additional firms.
5. Typically, the possibility of excessive entries resulting in low accumulated volumes per firm and high industry costs does not materialize.

Spence also looks at the two period model in which open and closed loop solutions can be calculated. He gets similar results as in the previous model. Spence also notes that there is another aspect of the study of learning curves: the interdependency of curves. The curves are not necessarily unique for each firm. They transfer from one firm to another through the hiring of competitors' employees or other channels. This causes a decline on return to investment in accumulated volume, but this also reduces industry-wide costs. Spence shows these results numerically for the two period case while allowing for interdependent learning curves.

The importance of this research is that it is one of the few theoretical studies where learning is an important factor in determining the firm's output and pricing decisions. In particular, it may be the only article that examines the industry-wide equilibrium associated with firms where the production process is characterized by significant learning.

Brueckner, J. K. and Raymon, N. (1983)

Brueckner and Raymon develop a model in a continuous time framework when production cost is a decreasing function of accumulated output (learning by doing). The authors examine optimal production plans under monopolistic and competitive markets, and they derive a model of the socially optimal production, i.e., a plan where a welfare measure is equal to consumer and producer surplus.

For the first model, monopoly, $p(x,t)$ is the demand curve with $\partial p/\partial x < 0$. The profit from an output, x, when volume is v, is

$$\pi(x,v,t) = xp(x,t) - C(x,v,t). \tag{2.30}$$

Noting that $x = dv/dt = \dot{v}$, the objective function is

$$\max \int_0^T \pi(\dot{v},v,t)\, e^{-rt} dt, \tag{2.31}$$

which is a calculus of variations problem.

The necessary conditions include the Euler equation,

$$\frac{\partial\pi}{\partial v} e^{-rt} - \frac{d}{dt}\left(\frac{\partial\pi}{\partial\dot{v}} e^{-rt}\right) = 0, \tag{2.32}$$

and the transversality condition,

$$\frac{\partial\pi}{\partial\dot{v}}\Big|_{t=T} = 0. \tag{2.33}$$

Knowing that $\partial\pi/\partial\dot{v} = p(\dot{v},t)+\dot{v}p(\dot{v},t)-C(\dot{v},v,t)$, the above may be written as

$$\frac{d}{dt}[MR(t) - MC(t)] - r[MR(t) - MC(t)] = MBL(t), \tag{2.34}$$

and

$$MR(T) - MC(T) = 0. \tag{2.35}$$

This follows from the fact that $p(\dot{v},t) + \dot{v}\, p(\dot{v},t)$ is marginal revenue at t for a given production plan or MR(t), and $C(\dot{v},v,t)$ is marginal cost at t or MC(t). MBL(t) is the marginal benefit from learning.

Equations (2.34) and (2.35) indicate that before the end of the production period, marginal cost overstates the incremental cost of an extra unit since the learning effect is not considered. At the end of the period, however, MC correctly measures incremental cost, so $MR(t) - MC(t) = 0$ is a necessary condition.

In the perfect competition case, the authors let $s(t)$ be the independent finite price path for each firm. The maximand then is

$$\int_0^T \Phi(\dot{v},v,t) \quad e^{-rt}dt \tag{2.36}$$

where $\Phi(.) = s(t)\dot{v} - C(.)$.

Noting $\partial\Phi/\partial\dot{v} = s(t) - MC(t)$, the conditions analogous to equations (2.34) and (2.35) are

$$\frac{d}{dt}[s(t) - MC(t)] - r[s(t) - MC(t)] = MBL(t), \tag{2.37}$$

and

$$s(T) - MC(T) = 0. \tag{2.38}$$

In this case, the equilibrium price path does not exist. Technically, this shows that the cost function is not convex. When this is the case, the perfect competitor's optimization problem has no solution. The incompatibility between increasing returns and perfect competition extends to the case of constant returns when production involves learning. Moreover, decreasing returns to scale, or convex C, does not guarantee optimality.

Brueckner and Raymon derive the same kind of model for the socially optimal case and conclude that as in monopoly production, socially optimal production plans call for a constant output rate. However, a monopolistic firm produces less than the socially optimal firm throughout the production period.

Womer, N. K. (1979)

The model developed in this paper combines a neoclassical production function with a learning hypothesis. Prior to this paper, both concepts had been used in empirical studies of production, but the integration of the two approaches had not been successful. The attempts were mostly conceptual, rather than being analytical.

It is assumed that the firm is engaged in production to order. This order specifies a quantity to be produced and a delivery date for output. The model uses a production function to relate output rate to a class of inputs whose use rates can be varied over the production period. Resources with fixed use rates are not considered in this class. Since the model assumes that the relative prices of resources do not change, the entire class may be represented as a composite resource. This class is considered as a variable input where all other inputs are assumed to be fixed. The following notation is needed to define the model:

$q(t)$ = the output rate on the program at time t,

$x(t)$ = the variable composite resource use rate at time t,

$Q(t) = \int_0^t q(\tau)d\tau$, cumulative production experience at time t,

δ = a parameter describing learning,

γ = a returns to the variable resource parameter,

C = discounted variable program cost measured in labor units,

T = the time horizon for the production program,

ρ = the discount rate,

V = the planned volume of output to be produced by time t.

The production function relates output rate, $q(t)$, to both resource use rate, $x(t)$, and cumulative production experience, $Q(t)$. Any resources required for induced production rate changes are assumed to be included in $x(t)$. The production function is of the form

$$q(t) = AQ^{\delta}(t)x^{1/\gamma}(t). \tag{2.39}$$

This function has two notable characteristics:

1. The production function is homogeneous of degree $1/\gamma$ in the resources. It is assumed that $\gamma > 1$; that is, there are diminishing returns to the variable composite resource. This implies that more resources are needed to maintain a constant output rate as $Q(t)$ increases.
2. Production experience induces neutral technological change in production process. This simplifies the analysis considerably. Otherwise, the impact of experience on the use of each resource must be specified.

Assuming constant relative prices, discounted program cost can be written in units of the composite resource as

$$C = \int_0^T x(t)e^{-\rho t}dt. \tag{2.40}$$

For a firm that has contracted to produce V units by time T, the problem is

$$\text{Min} \quad \int_0^T x(t)e^{-\rho t}dt. \tag{2.41}$$

s.t.

$$q(t) = AQ^{\delta}(t)x^{1/\gamma}(t),$$

$$x(t) \geq 0,$$

$$Q(0) = 0, \tag{2.42}$$

and

$$Q(T) = V.$$

This problem can be solved by optimal control theory, but a simple transformation permits a solution using the calculus of variations. The solution to the model yields the following optimal time paths for cumulative output and output rate:

$$Q(t) = V\{[e^{\rho t/(\gamma-1)}-1]/[e^{\rho T/(\gamma-1)}-1]\}^{1/(1-\delta)}, \tag{2.43}$$

and

$$q(t) = V[e^{\rho T/(\gamma-1)}-1]^{1/(1-\delta)}[\frac{\rho}{(1-\delta)(\gamma-1)}]e^{\rho t/(\gamma-1)} [e^{\rho t/(\gamma-1)}-1].^{\delta/(1-\delta)} \tag{2.44}$$

Both of these functions increase at an increasing rate with respect to time throughout the program. They imply that at any given time, both cumulative output and output rate decrease as T increases for fixed V. This result is expected since the firm has more time to produce the same amount of output.

Womer derives the following expression for the total discounted cost for a production program defined by V and T:

$$C(V,T) = [\frac{\rho}{(\gamma-1)}]^{(\gamma-1)} (1-\delta)^{-\gamma}A^{-\gamma}V^{\gamma(1-\delta)}[e^{\rho T/(\gamma-1)}-1]^{1-\gamma}. \tag{2.45}$$

This function describes the planning situation since cost is a function of the planned quantities, V and T. This function explains the behavior of cost if the time path of production rate is optimal. Volume affects cost through the coefficient $\gamma(1-\delta)$. Learning (δ) has a positive impact since, as volume increases, learning increases and cost decreases, but the factor returns parameter (γ) has a negative impact. Therefore, the net effect depends on the magnitudes of these two parameters.

The time path of cumulative discounted cost for given V and T is given by changing the upper limit of integration from T to t where t<T. Thus:

$$C(t|V,T) = [\frac{\rho}{(\gamma-1)}]^{(\gamma-1)}(1-\delta)^{-\gamma}A^{-\gamma}V^{\gamma(1-\delta)}\{\frac{e^{\rho t/(\gamma-1)}-1}{[e^{\rho T/(\gamma-1)}-1]^{\gamma}}\}. \quad (2.46)$$

This function describes the production situation since V and T are both fixed and cost is only a function of time.

Womer also compares three production programs with the same volume but at different time horizons. Even though discounted cost increases at an increasing rate in each case, total discounted cost decreases for the longer time horizon. If equation (2.42) is solved for t and the result is substituted into equation (2.46), an alternative formulation for the production situation is obtained: i.e.,

$$C(t|V,T) = [\frac{\rho}{(\gamma-1)}]^{(\gamma-1)}(1-\delta)^{-\gamma}A^{-\gamma}V^{(\gamma-1)(1-\delta)}$$

$$[Q(t)]^{(1-\delta)}[e^{\rho T/(\gamma-1)}-1]^{1-\gamma}. \quad (2.47)$$

This function states that cumulative output affects cost only through the learning parameter, (δ), while volume affects cost through the joint effects of learning, (δ), and the factor returns parameter, (γ).

If the assumption of negligible variations in relative resource prices holds, the above cost function is consistent with an empirical learning curve which relates cumulative discounted cost to cumulative output. However, in the learning curve literature, references to discounted cost are not common; therefore, differences should be expected between learning curves estimated by the above model and those estimated using undiscounted cost data.

Learning curves are popular descriptors of cost on aircraft programs. However, their use is not always consistent with either the model or the estimating situation described by Womer. Learning curves estimated as a function of cumulative output are used to evaluate plans that relate cost to volume without regard to the factor returns parameter. The implication is that learning curves estimated using this methodology can be misleading. Since volume and time are not fixed for different programs, empirical learning curves that do not consider these variations do not illuminate the tradeoffs available. Womer demonstrates this by comparing three different production programs. Discounted cumulative average cost is plotted against the number of units produced for all three programs. The slopes of the curves are the same, but the first-unit cost for each program is different. The same plot also shows the negative effect of starting a production program with a large planned volume, and then stopping production before the target, V, is reached.

The author also shows the effect of changing γ, the factor returns parameter, by comparing the same three programs with different γ values. For the production situation, the intercepts are different, while the slopes are the same. However, for the planning situation, decreasing returns were so strong that they overpowered the learning effect, causing an increase in discounted cost with an increase in V.

In this paper, unlike Alchian's (1959) model, production rate is a decision variable. The result is consistent with the finding that the effect of production rate on cost can not be distinguished from the effect of volume.

<u>Womer, N. K. (1981)</u>

In neoclassical production theory, the cost function relates the cost of production to the quantity of output produced per unit of time, and in some cases, to the prices of resources used in the production process. On the other hand, the learning curve relates cost to the number of items produced without considering flow concepts such as production rate or resource use rate.

As discussed in the earlier sections of this dissertation, there have been a number of attempts to integrate these two approaches. Washburn (1972) was the first researcher to formulate a model which related discounted profits to production rate and the cumulative number of items produced. In this paper, Womer extends Washburn's line of thinking and develops a model for a firm producing to an order which specifies a quantity and a delivery date for output.

The model augments a production function with a learning hypothesis, and relates output rate to two classes of inputs. The first input class is labor services, a composite input that is composed of resources whose use rate can be varied over the production period. The second input class, capital, is constant and is acquired prior to the start of production.

The variables of the model are

q = program production rate,

$\ell(t)$ = labor use rate at time t,

L = quantity of augmented labor,

K = quantity of capital,

$Q(t) = \int_0^t q(\tau)d\tau$, cumulative production experience at time t,

δ = a parameter describing learning,

γ = a returns to resource use parameter,

C = discounted program costs in labor units,

T = the time horizon for the production program,

V = the volume of output to be produced by time t,

p(t) = the daily unit cost of capital in labor units,

ε = the time elasticity of the cost of capital,

σ = the elasticity of substitution.

The production function has the following form:

$$q = L^{\delta}h(\frac{K}{L}). \tag{2.48}$$

This function relates the rate of labor use and the stock of capital to output rate. Learning enters the model as labor augmenting technological change. Thus,

$$L = Q^{\delta}(t)\ell(t) \tag{2.49}$$

for $0 < \delta < 1$. Also, q is constrained to be independent of t so that

$$q = V/T, \tag{2.50}$$

and therefore $Q(t) = qt$.

Since K and q are both constant, the production function implies that L is not a function of time. So after dividing equation (2.49) by q and rearranging:

$$\ell(t)/q = (L/q)\ Q^{-\delta}(t). \tag{2.51}$$

After substituting from equation (2.51), the following relationship is obtained:

$$\ell(t) = Lq^{-\delta}t^{-\delta}. \tag{2.52}$$

This relationship states that the rate of labor use falls with time because of learning and a constant output rate.

The daily unit cost of capital, P, is assumed to be

$$P(t) = pt^{-\varepsilon} \tag{2.53}$$

where $0 < \varepsilon < 1$, and p is the unit cost of capital for day one.

The unit cost of capital and program costs are measured in labor units so the wage rate is implicit. Also, the discount rate is restricted to one by assuming large time units. The present value of the cost stream is written as

$$C = \int_0^T [\ell(t) + Kpt^{-\varepsilon}]\ e^{-t}dt, \tag{2.54}$$

$$= Lq^{-\delta}\int_0^T t^{-\delta}e^{-t}dt + Kp\int_0^T t^{-\varepsilon}e^{-t}dt. \tag{2.55}$$

The above is the sum of two gamma functions. The first integral is $\Gamma(1-\delta,T)$, and the second is $\Gamma(1-\varepsilon,T)$; so the firm's problem is

$$\text{Min} \quad C = Lq^{-\delta}\Gamma(1-\delta,T) + Kp\Gamma(1-\varepsilon,T) \tag{2.56}$$

$$\text{s.t.} \quad q = L^{\gamma}h(\tfrac{K}{L}). \tag{2.57}$$

However, q is determined by the prior specified values of V and T; that is, q is not a decision variable.

Womer solved this problem and derived the following cost function:

$$C = \gamma q^{1/\gamma}p\Gamma(1-\varepsilon,T)/h'. \tag{2.58}$$

When T is fixed, an increasing production rate can affect program costs in two ways. First, cost may increase at an increasing rate because of diminishing returns to the variable resources. Second, cost may fall because of a reduction in the price of effective labor and a decreasing capital-labor ratio which increases h. The net effect of a production rate change on costs can be found by comparing the relative magnitude of these effects.

With q fixed, increasing V by increasing T also has two effects. Cost may be increased because of an increase in the effective price of capital, or cost may increase or decrease because of a changing effective price ratio. Again, the magnitude and the direction of the second effect determines the net impact of volume on cost.

Womer compared his results with the propositions put forward by Alchian and Hirshleifer. Only the propositions that are related to the above model are examined. The results are given in Table 2.1.

As is shown in the table, of the eight propositions, Womer's assumptions are sufficiently strong to satisfy Alchian's (1959) first, third, fourth, and fifth propositions and Hirshleifer's (1962) first proposition. The other three propositions failed to be satisfied. Alchian's assertion that marginal cost increases with production rate when V is fixed was found to require that the production function exhibit substantial decreasing returns to the variable factor if there were possibilities for input substitution.

Alchian's 'conjectural proposition' that marginal costs decreases with volume was found to hold in the presence of input substitution and substantial learning. Hirshleifer's proposition that marginal costs rise with production rate when the length of the program is fixed does not always hold in the presence of learning.

TABLE 2.1

Propositions and Model Results

Propositions	Sign Asserted	Model Results
	Alchian and Oi	
1. $\partial C(V/q)/\partial q$	+	+
2. $\partial^2 C(V,q)/\partial q^2$	+	+ if γ and σ both small
3. $\partial C(V,q)/\partial V$	+	+
4. $\partial^2 C(V,q)/\partial V^2$	-	-
5. $\partial [C(V,q)/V]/\partial V$	-	-
6. $\partial^2 C(V,q)/\partial V \partial q$	-	- if σ is large and $\delta > \varepsilon$
	Hirshleifer	
1. $\partial C(T,q)/\partial q$	+	+
2. $\partial^2 C(T,q)/\partial q^2$	+ if q is large	+ if γ and σ both small

The author's conclusion is that in order to specify the form of the cost function, more empirical work is needed to estimate the function's parameters. Without this empirical work, the shape of the program cost function is indefinite.

Womer, N. K. and Gulledge, T. R., Jr. (1983)

This research is an extension of the work that was presented by Womer (1979). As in the previous model, this model assumes that both learning and production rate have an impact on total program cost.

It is assumed that costs are affected by four major determinants which are called cost drivers. The first cost driver is learning by doing. Learning affects cost by influencing efficiency at each position on the production line. This implies that experience on the production line is always changing with time. The second cost driver is learning how to produce more efficiently over time. That is, early in the production period some labor is spent learning to be efficient. This implies that labor is more efficient at later positions than at early positions on the same airframe. The third cost driver is the speed of the production line. Increasing the speed of the line is expected to require more labor without considering the learning compensation. Furthermore, due to diminishing returns, the required additional labor is expected to be more than the increase in speed, proportionally. The fourth cost driver is the length of the production line. By adding more positions to the production line, the amount of work done per time period can be increased. The effect of this driver may be significant when changes occur in the production period as opposed to the planning stage. Costs increase because of crowded facilities, overused tools, or inefficient use of fixed resources.

As in Womer's (1979) model, the homogenous production function is augmented with a learning hypothesis. The model also includes all the previously mentioned cost drivers. The following notation is needed to define the model:

i = an index for a batch of airframes in the same lot, j, all of which are to be delivered at time t_{ij},

n_j = the total number of batches in lot j,

m = the total number of lots in the production program,

D_{ij} = the number of airframes in batch i of lot j,

E_{ij} = a measure of experience prior to the midpoint of batch i; that is,

$$E_{ij} = \sum_{k=1}^{j-1} \sum_{h=1}^{n_k} D_{hk} + \sum_{h=1}^{i-1} D_{hj} + \tfrac{1}{2}D_{ij},$$

t_j = the date of work begins for all the batches in a given lot, j,

t_{ij} = the date that work ends for batch i of lot j,

$Q_{ij}(t) = \int_{t_i}^{t} q_{ij}(\tau)\, d\tau$, i.e., cumulative output at time t,

$x_{ij}(t)$ = the rate of resource use at time t on batch i of lot j,

δ = a learning parameter prior to batch i,

ε = a learning parameter on batch i,

γ = a returns to resource use parameter,

α = a parameter related to decreases in labor productivity that occur toward the completion of the program,

η = a parameter describing returns to the size of the batch,

ν = a parameter describing returns to the length of the production line,

C = discounted variable program cost,

and

C' = discounted variable costs for a single batch of airframes.

The production function is

$$q_{ij}(t) = AV^{\nu}D_{ij}^{\eta}E_{ij}^{\delta}Q_{ij}^{\varepsilon}(t)(t_{ij}-t)^{\alpha}x_{ij}^{1/\gamma}(t). \qquad (2.62)$$

This function accommodates the fact that the nature of work varies from position to position along the production line, as well as including the cost drivers.

The term E_{ij}^{δ} describes learning-by-doing in the production of a given batch i in lot j. The terms $Q_{ij}^{\varepsilon}(t)$ and $(t_{ij} - t)^{\alpha}$ describe the learning that occurs during the production of that specific batch. All three parameters (α, δ, and ε) are expected to be between 0 and 1.

The terms D_{ij}^{η} and $x_{ij}^{1/\gamma}(t)$ explain the effect of the speed of the production line. In these terms, η is expected to be between 0 and 1 while γ is greater than 1. The term V^{ν} captures the effect of assembling alternative numbers of airframes in the same facility. The parameter ν is expected to be negative and small.

The problem may be stated as

$$\text{Min} \quad C = \sum_{j=1}^{m} \sum_{i=1}^{n_j} \int_{t_j}^{t_{ij}} x_{ij}(t)e^{\rho t}\, dt \tag{2.63}$$

s.t

$$\begin{aligned} q_{ij}(t) &= AV^{\nu}D_{ij}^{\eta}E_{ij}^{\delta}Q_{ij}^{\varepsilon}(t)(t_{ij}-t)^{\alpha}x_{ij}{}^{1/\gamma}(t), \\ Q_{ij}(t_{ij}) &= D_{ij}, \qquad i = 1,2,\ldots,n_j, \\ Q_{ij}(t_j) &= 0, \qquad j = 1,2,\ldots,m. \end{aligned}$$

Since total cost is monotone nondecreasing and the subproblems are additive, the solution can be obtained by minimizing each of the subproblems. The objective function can be written as

$$\text{Min } C' = \int_{t_i}^{t_{ij}} x_{ij}(t)e^{-\rho t}dt. \tag{2.64}$$

This is an optimal control problem that can be solved by minimizing the Hamiltonian function. The authors used a transformation similar to that used by Womer (1979) and solved the model by using classical variational techniques. The optimal time path of resource use was found as

$$x_{ij}(t) = BV^{-\gamma\nu}E_{ij}^{-\gamma\delta}D_{ij}^{\gamma(1-\varepsilon-\eta)}\Gamma^{-\gamma}[\rho(t_{ij}-t)/(\gamma-1),\alpha\gamma/(\gamma-1)+1]$$
$$(t_{ij}-t)^{\delta\gamma/(\gamma-1)}e^{-\gamma\rho(t_{ij}-t)/(\gamma-1)}. \qquad (2.65)$$

However, since aircraft data is usually quarterly, the quantity of interest is the total resource use over a quarterly period. Assuming that T_k and T_ℓ are the beginning and the ending dates for the quarterly period of batch i, the quarterly resource requirement function may be written as

$$X(T_k) - X(T_\ell) = \int_{T_\ell}^{T_k} x(t)dt, \quad \text{or}$$

$$X_{ij}(T_k)-X_{ij}(T_\ell)=B'E_{ij}^{-\delta\gamma}D_{ij}^{\gamma(1-\varepsilon-\eta)}$$
$$\Gamma^{-\gamma}[\rho(t_{ij}-t_j)/(\gamma-1),\alpha\gamma/(\gamma-1)+1]V^{-\gamma\nu}$$
$$\{\Gamma[\gamma\rho(t_{ij}-T_\ell/(\gamma-1),\alpha\gamma/(\gamma-1)+1]-$$
$$\Gamma[\gamma\rho(t_{ij}-T_k)/(\gamma-1),\alpha\gamma(\gamma-1)+1]\}. \qquad (2.66)$$

However, because of the nature of the data, it is impossible to observe the above quantity. Only direct man-hours per lot is observable, therefore the quarterly resource requirement function must be summed over all batches in the lot, i.e.,

$$\sum_{i=1}^{n_j} [X_{ij}(T_k) - X_{ij}(T_\ell)]. \qquad (2.67)$$

To explore the applicability of the theory, the parameters in equation (2.66) are estimated using data on the C141 airframe program. Nonlinear least squares is used, and all of the estimates are significantly different from zero. The signs of the estimates also agree with a priori expectations. The values of the parameters are consistent with the theory, i.e., $\gamma > 1$ and $\delta = .28$ which implies an 83% learning curve.

Womer and Gulledge plotted the predicted time path of resource use for the program and the actual resources used. The fit of the model is satisfactory ($R^2 = .69$) even though the model variation with time is greater than exhibited by the data. The reason is that the model does not include hiring and firing costs.

The authors also performed sensitivity analyses on the solution. The most important analysis involved checking the effects of exogenous delivery schedule changes. First, they examined the case where the first airframe in the program was delivered one month later. The net change is a small increase in predicted program cost. A second sensitivity demonstrated the effect of delivering the last airframe one month earlier. This reduces the time to work on the last airframe, so learning is reduced and V also increases for that period. Since the last airframe combines with the previous batch, the total number of batches decreases, and this offsets the increases due to other effects. The net effect will be a slight decrease in cost.

Next, the authors consider the advancement of the delivery of one airframe in the middle of the program. Learning decreases, but V also decreases. The net effect is a uniform decrease in the path of resource use. The net effect of lot release date changes is also examined. This also reduces cost slightly.

This study provides a theoretical model with parameters which are estimated from actual data. The estimation results and the data are compared, and sensitivity analysis is performed on the delivery schedule of the model.

Concluding Remarks

With the introduction of Wright's (1936) seminal work, a new era was initiated in empirical cost studies. Since then, many attempts have been made to integrate the engineering approach to cost estimation with economic theory. However, most of these studies either stopped short of producing empirical results or produced these results based on unrealistic assumptions or simplifications. Although this subject area has been attracting substantial attention for years, there is still a remarkable shortage of literature, both in theoretical and practical applications. This dissertation is aimed to fill some of these gaps by using the results that were obtained by the above mentioned researchers.

CHAPTER III

REPRICING AIRCRAFT PROCUREMENT PROGRAMS

Planned annual procurement quantities for defense weapon systems are often changed after the production program has started. Therefore, researchers have developed methods for repricing ongoing aircraft procurement programs. The problem has been studied by many researchers [e.g. Bemis (1981, 1983), Bohn and Kratz (1984), and Smith (1976)], but the research gains have been painfully slow. One successful application was presented by Balut, Gulledge, and Womer (1986).

In this chapter, the popular methods for repricing aircraft procurement programs are reviewed. These methods are basically of two types. The first type of models are classified as "regression methods." These methods purport to consider production rate changes, but as will be demonstrated, they do not. The second group of models is classified as "learning curve methods." These models do not consider within-lot production rate effects. The assumption of these models is that within-lot rate effects are relatively insignificant. The rate effect is assumed to be dominated by other factors, e.g., the reallocation of in-plant fixed costs after a change in procurement quantity. The importance of the within-lot production rate effect is examined in this dissertation. Therefore, the background material in this chapter is central to a proper understanding of the analysis in later chapters.

Regression Models for Repricing Procurement Programs

The primary regression relationship as presented by Ayres, et al. (1984) is

$$z = AX^{B}Y^{C} \tag{3.1}$$

where

z = the unit cost of the Xth item produced,

A = a constant referred to as the surface initialization point,

X = cumulative quantity produced,

B = an exponent which describes the slope of the quantity/cost curve,

Y = a proxy for the production rate in effect,

C = exponent which describes the slope of the rate/cost curve.

The basic idea in using this model to reprice procurement programs is to use (3.1) as a prediction equation. Under the assumption that lot size is a good proxy for production rate, and cumulative lot size is a good proxy for cumulative quantity produced, then the unit cost of any hypothesized change in lot size can be predicted from equation (3.1). The approach seems simple enough, but further analysis uncovers problems.

After examining the literature, it seems appropriate to note the inconsistencies in the variable definitions associated with equation (3.1). First, consider the definition of unit cost. Smith (1976, p. 37) measured cost in direct labor hour units; i.e., z = the average number of direct labor hours required to manufacture each pound of airframe. The reason for this assumption is that the different components of cost do not follow the same learning curve. This result was noted by Asher (1956, p. 111). As Asher notes, an approach that is often used is to exclude all non-recurring costs and construct a composite curve using only recurring costs. Asher (1956, p. 111) also notes that "if the curves for the various elements of airframe cost differ in slope from one another, then a linear curve may not be an accurate representation of the composite curve." It does not seem reasonable to expect that airframe, propulsion, electronics, armament, and other costs should follow the same learning curve, but the magnitude of the error from using a composite curve is system dependent. Smith avoided the problem by examining direct labor requirements.

Bemis (1981, 1983) also uses equation (3.1) to predict unit costs for several weapon systems programs. In these papers, z is defined as unit cost; but the exact definition of unit cost is unclear. The discussion (1981, p. 85) leads one to believe that the model is used to predict total system unit costs. If this is the case, the analysis is probably incorrect; that is, there is no reason to expect all components of total cost to follow the same learning curve.

Cox and Gansler, (1981) in their study of quantity, rate, and competition, define unit cost as "cost to the government, which corresponds to price in classical economics." However, price includes the allocation of fixed overhead. While the fixed overhead cost per unit may decline as the number of units increase, there is no reason for that portion of cost to follow the learning curve. The allocation of overhead varies from lot to lot which can cause upward or downward shifts in price in the presence of learning.

Apparently, most regression models that apply to aircraft are used to project costs at the unit recurring flyaway level. This is not correct if the variable portion of indirect costs is not included and the fixed portion is not excluded, but it is difficult to assess the error implications. Clearly, the error is dependent upon the amount of fixed indirect costs on the program of interest. Balut (1981) is correct in noting that only the variable part of costs should be projected with the mathematical model. This would involve a regrouping of costs. The variable part of overhead cost should be separated from the fixed portion, and costs should be classified as fixed and variable. The variable costs should be modeled as a function of program variables, and the fixed cost should be considered separately. Even this approach is an approximation since this assumes that all variable costs can be modeled with the same relationship. A more appropriate formulation would require modeling each component of variable costs (e.g., material, labor, etc.) with a separate relationship, and summing the projections to obtain an estimate for total variable costs. Note that variable costs as defined here contain direct costs plus the portion of indirect costs that are variable. A discussion of the cost separation procedure is presented by Balut (1985) and Balut, et al. (1986).

Much of the controversy surrounding the use of regression models [e.g., equation (3.1)] is a direct result of a different dependent variable, z, from one study to another. One should expect conflicting results when in some studies z is variously defined as direct man hours, total variable cost, total recurring cost, and price. Each of the quantities is very different. The point of this discussion is that researchers should be explicit when defining unit costs, and much additional research is needed in the area of cost aggregation.

A second definitional concern is related to the definition of production rate. Since data are not usually compiled on annual production rate, annual procurement rate is often substituted as a proxy. That is, the variable Y in equation (3.1) is set equal to the annual procurement lot quantity. Again, the magnitude of this error is difficult to assess as it is program dependent. However, it is believed that the error can be significant for those programs with long production profiles such as aircraft. More precisely, aircraft that are procured in a given year are usually produced over a period of four to six years. To contain a measure of annual production rate, the procurement lot quantity must be allocated to years in which the actual production occurred. Smith [(1976), p. 41] used what he called the "lot average manufacturing rate", the number of airframes in a production lot divided by the production time span. This assumes a uniform rate distribution, a result that is inconsistent with knowledge of the production process. Still, this approach is probably better than using lot size as a proxy for production rate.

It is not clear how production rate is measured in some regression models. Production rate is not clearly defined by Ayres, et al. (1984) or Bohn and Kratz (1984) but Cox, et al. (1981, p. 4-13) state that "lot size was used as a proxy for production rate with the exception of a few instances where it was known the quantity was produced over more than one year." No explanation is provided for the programs with production periods that exceed a year. In Bohn and Kratz [(1984), pp. 4-5 to 4-6], the discussion leads one to believe that annual procurement lot quantity is being used as a proxy for production rate. This is confirmed in Bolton's (1985) thesis where he examines equation (3.1). He states "the only thing it [equation (3.1)] requires which the standard learning curve formulation does not is a production rate and it is convenient to use readily available buy schedules as a proxy for this."

It is noted that this problem has been addressed by Balut, et al. (1986) and research is continuing in this area. However, since this research relates to production rate, it is important to discuss the magnitude of error introduced by the "lot quantity" proxy. Intuitively, the error is expected to be large if annual procurement quantities are fluctuating and the production period is lengthy.

It is also noted that the choice of an equivalent production rate measure also influences the measure of cumulative production. Cumulative annual procurement quantity is not appropriate. For example, suppose the 1987 buy quantity for some system is 400 and the production period for these 400 units is four years. Cumulative production through 1987 cannot include all of these 400 units since all 400 units have not been produced at the end of that procurement year. Some units will be produced over the remainder of the four year production period.

Parameter Estimation

It has been asserted that a major advantage of some regression models is that more accurate results are achieved because the model's parameters are estimated by nonlinear techniques [see, for example Ayres, et al. (1984) or Bohn and Kratz (1984)]. In addition, it has also been asserted that the nonlinear approach reduces the multicollinearity between cumulative output and output rate. These issues are addressed in this section.

The accuracy question only indirectly relates to the estimation technique. The problem follows from the fact that the estimation of the parameters in equation (3.1) is complicated because data are not available by production unit. Data are only available by procurement lot. If unit data were available, equation (3.1) could be estimated by linear or nonlinear regression with comparable results (Graver and Boren, 1967).

The problem with lot data is really not a problem of the learning curve formulation. It is indeed a data problem, a problem that follows from not knowing the true lot midpoints without first knowing the learning curve slope. This problem is discussed by Gallant (1968).

Some regression modelers avoid the estimated midpoint bias by using the "Boeing" approximation presented by Asher [(1956), pp. 35-36]. This is an approximation for total cost when the unit variable cost function is a learning curve. Total cost is approximated with the following integral:

$$TLC_i = \int_{N+.5}^{N+K_i+.5} AX^B Y^C \, dX \qquad (3.2)$$

where N = cumulative lot quantity, K_i = the number of units of lot i, and TLC_i is the total cost of lot i.

For the integration of equation (3.2) it is assumed that Y is constant. The resulting expression is

$$TLC_i = \frac{A}{B+1}\,[(N + K_i + .5)^{B+1} - (N + .5)^{B+1}]Y^C. \tag{3.3}$$

This result [equation (3.3)] is the area under a learning curve; a learning curve with slope parameter B and first unit cost AY^C. That is, the approximation assumes that production rate proxy is constant. However, for the parameter estimation in equation (3.3), production rate proxy is allowed to vary.

This approximation is difficult to interpret when one considers the fact that the time required to produce a lot of aircraft may exceed four years. Since equation (3.3) assumes that production rate is constant over the time required to produce the lot, this implies production rate is constant over a number of years. Since the variable Y changes from year to year, an inconsistency results.

Bolton (1985), apparently following earlier work by Bohn and Kratz (1984), used the following approximation of total lot cost:

$$TLC_i = \int_N^{N+K_i} AX^B Y^C dX = \frac{A}{B+1}\,[(N+K_i)^{B+1} - N^{B+1}]Y^C. \tag{3.4}$$

Unfortunately, this can be a very poor approximation. An error analysis of this unit learning curve approximation (in the absence of the production rate effect) can be found in Camm, et al. (1987b). Both of the integral approximations used in equations (3.2) and (3.4) exceed actual cost. However, the perturbed approximation used in equation (3.3), where the integral units are perturbed by .5, is a much better approximation than that used in equation (3.4). It is clear from

Figures 3.1 and 3.2 why this is the case. In Figure 3.1, obtained from equation (3.4), the shaded area represents overestimation of the actual lot cost (sum of the rectangles). The perturbed approximation is shown in Figure 3.2. It performs better than the unperturbed approximation because, as shown in Figure 3.2, the perturbed integral approximation contains some overestimation and some underestimation for each unit in the lot. The underestimation for each unit cancels some of the overestimation, yielding a net overestimation of total lot cost which is smaller than that obtained from the unperturbed approximation. In the worst case analysis presented in Camm, et al. (1987a), the perturbed approximation overestimated cost by 3% only while the unperturbed approximation overestimated by 290%! The degree of overestimation of the total lot cost is a function of where the lot occurs on the learning curve and the slope of the learning curve; the unperturbed approximation can induce serious error. The reader is referred to Camm, et al. (1987a) for a more detailed discussion of this error.

The functional form of the approximation used in the nonlinear least squares model can have a serious impact on the parameter estimates as shown below. Lot data for the C141 airframe program is used to illustrate this sensitivity. The parameters of the learning curve (in the absence of the rate effect) are estimated using three different models: a direct model using the summation over all units in the lot, a model using the unperturbed integral approximation of lot cost, and a model using the perturbed approximation of total lot cost:

$$\text{Model 1:}\quad \text{Min} \sum_{i} \left(y_i - \sum_{X=N+1}^{N+K_i} AX^B\right)^2 \qquad (3.5)$$

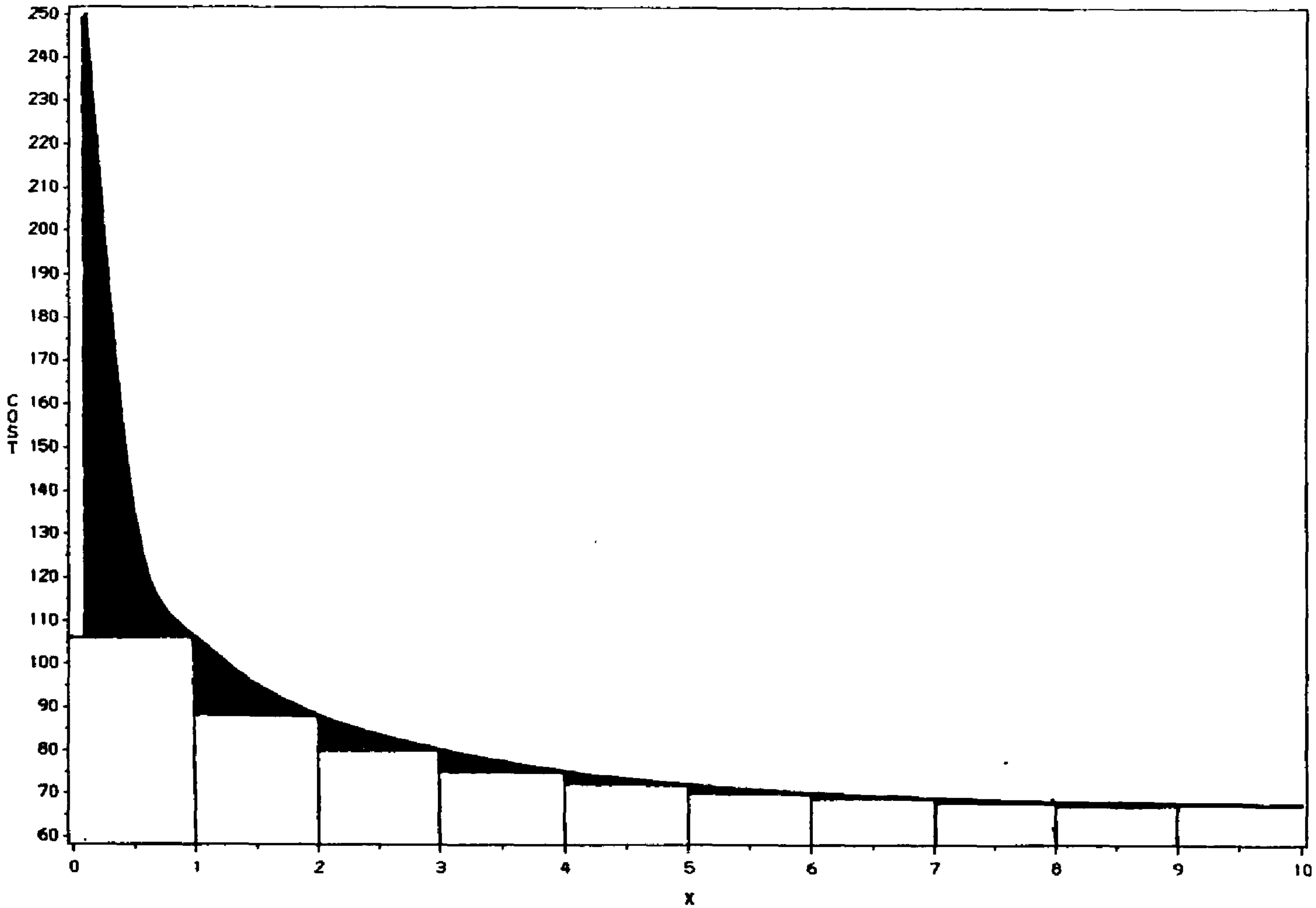


Figure 3.1 The Unperturbed Approximation.

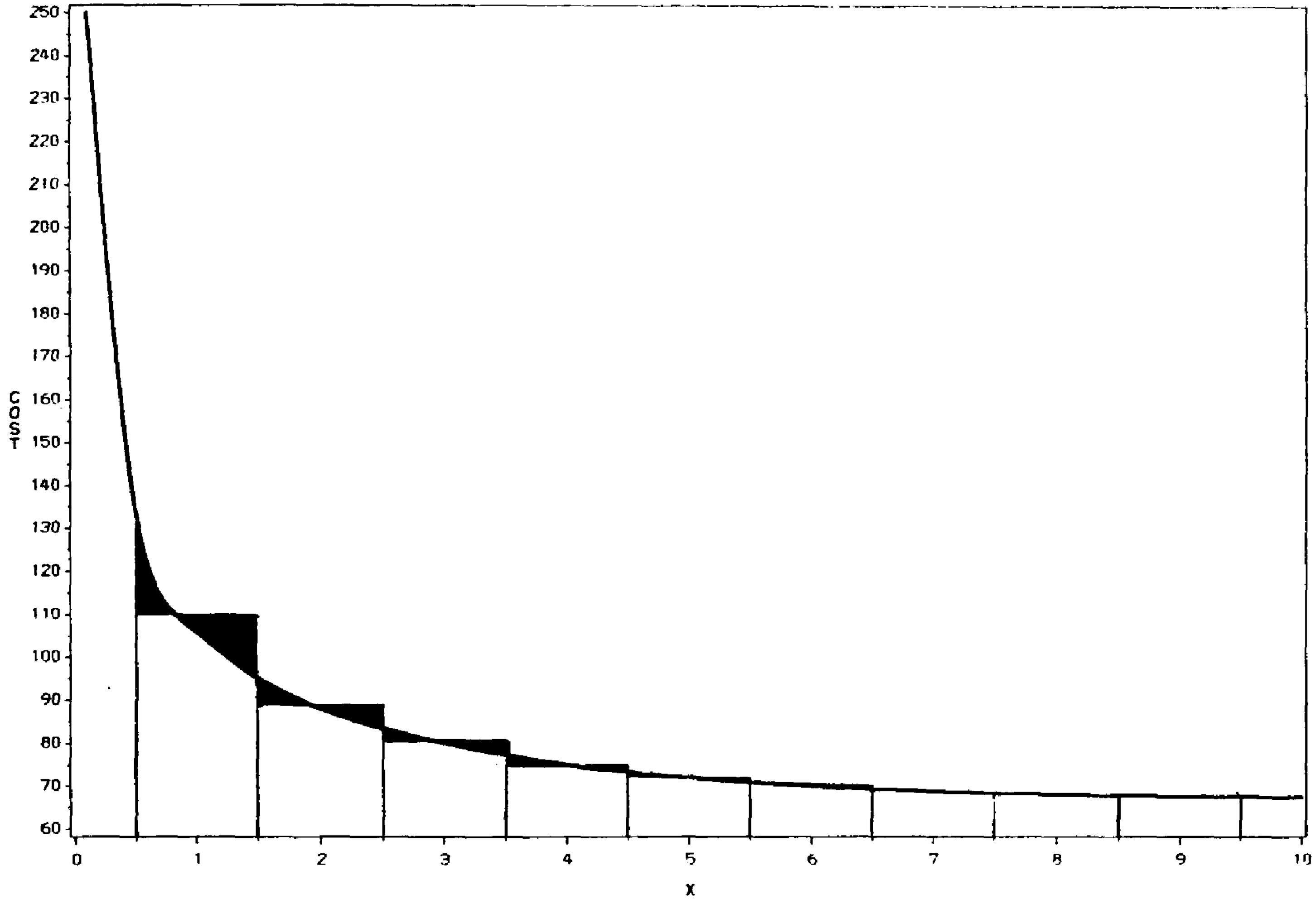


Figure 3.2 The Perturbed Approximation.

$$\text{Model 2:} \quad \text{Min} \sum_i (y_i - \frac{A}{1+B} [(N+K_i)^{B+1} - N^{B+1})]^2 \qquad (3.6)$$

$$\text{Model 3:} \quad \text{Min} \sum_i \{y_i - \frac{A}{1+B} [(N+K_i+.5)^{B+1} - (N+.5)^{B+1}]\}^2 \qquad (3.7)$$

where y_i = direct labor hours for the lot i and the remaining variables are as previously defined. The least squares results appear in Table 3.1.

Table 3.1

Model Comparisons

Model	$\hat{A}$	$\hat{B}$	Residual Sum of Squares (RSS) *
1	604,393	-.3897	492.2×10^9
2	521,025	-.3582	611.0×10^9
3	600,710	-.3884	492.2×10^9

*RSS were calculated after the estimation using the estimated parameters and model 1.

The results are exactly as anticipated; that is, the perturbed approximation is a good approximation of model 1, whereas the unperturbed is a poor approximation. The unperturbed approximation results in very different parameter estimates. The first unit cost, A, seems to be particularly sensitive to the use of the unperturbed approximation used in model 2. This is not surprising considering Figure 3.1. The area under the curve early on the learning curve contributes heavily to the overestimation of total lot cost inherent in the approximation. Consequently, the least squares estimate for the first unit cost is adjusted downward to offset the inherent overestimation of total lot cost. Furthermore, model 2 results in an overall poorer fit to the data (the RSS increases by 24%; see Table 3.1).

Ayres, et al. (1984) used model 2 augmented for production rate, whereas later work uses the augmented model based on model 3 [see, for example, Bohn and Kratz (1984)]. Bolton (1985) apparently used an augmented model based on model 2. It is suspected that this may account for his inability to duplicate other researchers' parameter estimates for some programs [Bolton (1985), p. 30].

It is not clear why an approximation has to be used at all. The "true" model based on the unit learning curve is model 1. Model 1 is just as easily augmented for production rate and programmed in SAS as model 2 or model 3. If an approximation is going to be used, it should be the perturbed approximation.

As for multicollinearity, it is difficult to see how an application of nonlinear least squares to equation (3.3) can solve the problem. Multicollinearity is not a problem related to statistical methodology; it is a data problem. It follows from the fact that variables that are highly correlated do not explain independent variables when equation (3.3) is used in least squares estimation that multicollinearity is still present. In Ayres, et al. [(1984), p. 2-5] it is stated, "the problem of multicollinearity between cumulative quantity and production rate is avoided by the use of a nonlinear function; however, the dependency between the two variables is not assessed." The same statement is made by Bohn and Kratz [(1984), p. 4-5]. Also, Gardner

(1985) states that the issue would be considered in Bolton's (1985) thesis, but there is only a statement of the problem in that work.

The concern with multicollinearity is that it makes it impossible to assign a meaningful interpretation to the model's coefficients. These problems were addressed by Camm, et al. (1987a). It is well known that if the independent variable correlation structure is the same in the prediction period as in the estimation period, the model may still provide satisfactory predictions. Of course, this assumes that the joint range of the observed independent variables is not violated during the prediction period. However, Camm, et al. (1987a) argue that these relevant ranges are probably violated.

Hypothesis Testing

This topic is discussed since the issue is raised in several of the documents supporting the regression approach. In general, there is little evidence of any "goodness-of-fit" testing in any of the documents supporting the regression approach. This is probably appropriate, since historically sample sizes for weapon system procurement programs are usually small. Still, the issue is discussed since the avoidance of biased and consistent estimators is stated as an advantage of the regression approach [see, for example, Bohn and Kratz (1984), p. 2-4 and p. 4-2].

The results follow from the works of Meulenberg (1965) and Goldberger (1968) and apply to parameter estimation in multiplicative models, such as equation (3.1). The result is demonstrated with a learning curve, but the same result applies to general multiplicative functions. Assume the learning curve

$$z = AX^B \tag{3.8}$$

where the variables are as previously defined. For parameter estimation in equation (3.8), it is assumed that all nonquantifiable factors are contained in a disturbance term, u, that satisfies the following assumptions:

$$E(u_i) = 0,$$
$$E(u_i^2) = \sigma^2 = \text{constant},$$
$$E(u_i u_j) = 0 \quad i \neq j,$$
$$E(X_i u_i) = 0.$$

For hypothesis testing, it is assumed that the random variable u_i is independent and identically normally distributed. Equation (3.8) is usually restated as

$$z = AX^B u, \tag{3.9}$$

and logarithms of both sides are taken prior to estimation. The estimable function is

$$\ln z = \ln A + B \ln X + \varepsilon, \tag{3.10}$$

where $\varepsilon = \ln u$. The parameters in equation (3.10) may be estimated by ordinary least squares.

If the parameters are estimated directly from equation (3.9), the functional form is

$$z = AX^B + u. \tag{3.11}$$

The models in equations (3.9) and (3.11) are different because their error structures are different. If the normality assumption applies to both equations (3.9) and (3.11), then there is an inconsistency with equation (3.10); i.e., the disturbance term in equation (3.10) is lognormally distributed. Since all hypothesis tests are dependent on the normality assumption, usual tests of significance are affected.

In general, these issues are important, but it is believed they are relatively unimportant when the regression approach is applied to military procurement programs. This is because the sample sizes are so small. There is no evidence of any hypothesis testing in any of the model documentations reviewed for this dissertation, and the problem described above is only important if the model is subjected to hypothesis testing.

An Alternative Regression Formulation

In Bolton's (1985) thesis, two additional production rate variation models are examined. One formulation is

$$Z = AX^{B}Y^{C}R^{D} \tag{3.12}$$

where $R = Y_i/Y_{i-1}$, D is the "production rate change parameter", and the other variables are as previously defined. Bolton states that the idea behind this model "is that the change in production rate from one lot to the next is as important in explaining the impact of production rate on cost as is the rate itself" [Bolton (1985), p. 26]. One can only speculate on the motivation for this formulation. One possibility is that the model in equation (3.1) does not provide the proper response to changes in the production rate proxy.

There are at least two reasons why equation (3.1) may not provide the proper response. The first reason is related to the collinearity that is typically observed in the data. Many researchers have estimated parameters in models similar to equation (3.1). In every case, cumulative quantity always explains most of the variation in the dependent variable, while rate has a small statistical impact. This does not mean that production rate is an unimportant variable; it is just impossible to separate the rate effect using regression analysis. Consequently, since the addition of the rate variable leads to a small reduction in the error sum of squares, the rate variable has a small impact on the forecast function.

A second reason why the model may be insensitive to the production rate proxy relates to the proxy itself. As previously mentioned, the estimation procedure assumes that production rate is constant over the procurement lot. Since a procurement lot is produced over a number of years, this assumption is equivalent to assuming that production rate is constant over a number of years. That is, the functional form for estimation assumes constant production rate over the lot and thus over time while the data for the estimation are generated by programs where rate is varying.

Bolton's formulation [equation (3.12)] attempts to address these limitations by adding change in production rate as a variable. If the ratio, R, is statistically significant, it appears that the model is more responsive to rate changes. Unfortunately, the model in equation (3.12) is still plaqued by the previously discussed problems, namely multicollinearity and the assumption of constant lot production rate.

This observation raises an interesting question. Is it possible to construct an appropriate model within the format of equation (3.1)?

The answer to the above question is unknown, but it is known that there are methodological problems with the approach. The motivation for equation (3.12) was to modify a convenient model to accommodate the available procurement lot data. The correct approach requires constructing the theoretical model that actually generated the data, not to try to force some convenient available model on the data. This research is directed toward identifying a model that generates procurement lot data. That is, one of the problems with the regression approach is that a simple model that was originally defined for explaining unit direct labor hours is being forced upon the more complex lot cost problem.

The above modeling issues were the motivation for constructing a model of total lot cost. The next section discusses such a model, the model developed by Balut, Gulledge and Womer (1986). The model still does not consider within-lot production rate variations, but it is more correct than the regression approach in that it is a model that determines total lot cost using allocation principles similar to those used by cost accountants.

The Learning Curve Approach to Repricing Made-To-Order Procurement Programs

The Balut, et al. (1986) paper presents a method for repricing aircraft programs under a proposed change in procurement quantity. The basic modeling relationship is a learning curve, but all costs are not treated as variable. This is important since the fixed (relative to variable) portion of total costs has been increasing over time. This implies the performance of learning curve cost estimating methods will be degraded if applied to total cost. The Balut, et al. model explicitly considers the portion of the cost that is quantity invariant (i.e., fixed) with respect to the quantity produced.

The following assumptions apply to their model:

1. All direct costs are variable.
2. Overhead is allocated on the basis of direct cost.
3. Since the aircraft that are procured in a given year are produced over a number of years, an expenditure profile is introduced into the model for allocating the variable cost or price over these years. It is assumed that expenditure profiles are constant from year to year.

Since the data requirements for research on this subject area have been a problem over the years, the information required to implement this model is classified into three categories by the authors.

1. Plant related model inputs:
   - expenditure profiles for the programs,
   - methods to separate expenditure into fixed and variable components,
   - forecasts of annual plant-wide expenditures.
2. Program related variables:
   - historical cost experience and future projections that are associated with planned acquisitions.

3. Economic input data:

   - historical economic escalation indices,

   - forecasts of economic escalations.

In general, the above data items are usually available to Department of Defense analysts. More detailed data, such as production line data, are usually not available.

The model explains the prices of annual contracts for a product as a function of the quantities procured each year. The model is sensitive to changes in the procurement plan and explicitly considers the allocation of fixed costs. It uses traditional estimation theory to relate variable cost to quantity, and allocates the fixed costs to the contracts using an accounting-like procedure.

The model also introduces the concept of business base for those cases where the plant runs programs other than the program under consideration. This enables the model to allocate the fixed cost over all the production programs in the plant.

The following notation is needed to define the model:

t = the index for the years over which a program is procured,

j = $(1,\ldots,J)$ the index for the annual contracts on the program of interest, $j = 0$ is used to indicate all other contracts in the producer's plant,

P_j = the price of contract j,

D_{jt} = the direct cost incurred on contract j in year t,

V_{jt} = the variable cost on contract j in year t,

V_{ot} = the variable cost in year t associated with contracts other than the program under consideration,

M_{jt} = the overhead cost allocated to contract j in year t,

F_{jt} = the fixed portion of overhead cost allocated to contract j in year t,

H_{jt} = the variable portion of overhead cost allocated to contract j in year t,

B_t = the contractor's in-plant business base in year t,

A_t = the book value of the contractor's assets in year t,

X_i = the direct cost on unit i.

The price of an annual contract has two components: direct costs and overhead costs, i.e.,

$$P_j = D_{j.} + M_{j.} \tag{3.13}$$

where $\sum_{t=1}^{N} D_{jt}$ is written as $D_{j.}$. Note that the 'dot' notation means that D_{jt} has been summed over all the years for which contract j is in production. It is also noted that P_j is the only quantity that is observable in the data. Data are not available on fixed and variable costs by contract. The essence of the Balut, et al. research is that a model is constructed that explains P_j which uses only the previously mentioned data items.

The direct cost of the ith production unit on the contract is

$$X_i = ai^b \tag{3.14}$$

where a is the first unit cost and b is parameter that represents the rate of learning. Again, it is noted that X_i is not observable in the data. Therefore, the direct costs associated with contract j can be written as

$$D_j = a \sum_{Q_{j-1}+1}^{Q_j} i^b. \tag{3.15}$$

The overhead component, $M_{j.}$, of the total price is also composed of two parts, fixed and variable. It can be modeled for a given year t as

$$M_{.t} = \beta_0 + \beta_1 A_t + \beta_2 D_t \tag{3.16}$$

where $\beta_0 + \beta_1 A_t$ represents the fixed part ($F_{.t}$), and $\beta_2 D_t$ represents the variable part ($H_{.t}$) [Balut, (1985)]. These cost separation models are exogenous to the Balut, et al. model. These models were developed at the Institute of Defense Analyses using proprietary contractor data.

Since the model assumes that fixed overhead is allocated to a contract based on the variable cost associated with that contract,

$$F_{jt} = [\frac{V_{jt}}{V_{.t}}] F_{.t}. \tag{3.17}$$

Defining π_{jt} (the variable cost expenditure profile) as the fraction of variable cost associated with contract j in year t, it follows that

$$V_{jt} = \pi_{jt} V_{j.}, \tag{3.18}$$

and

$$D_{jt} = \pi_{jt} D_{j.}. \tag{3.19}$$

Some of the variable costs in the plant may be associated with other programs; therefore total in-plant variable cost in a given year is

$$V_{.t} = V_{ot} + \sum_{j=1}^{J} V_{jt}. \tag{3.20}$$

After making the necessary substitution and summing over t, the following expression is obtained for the fixed cost of contract j:

$$F_j = (1+\beta_2)D_{j.} \sum_t [\pi_{jt} F_{.t} / (V_{ot} + (1+\beta_2) \sum_{k=1}^{J} \pi_{kt} D_{k.})]. \tag{3.21}$$

The price of the contract j can be obtained by using the relation between overhead cost and its fixed and variable components, namely

$$M_{j.} = F_{j.} + \beta_2 D_{j.}. \tag{3.22}$$

Substituting equations (3.20) and (3.21) into equation (3.13), the following model of the price of contract j is obtained:

$$P_j = (1+\beta_2)D_{j.} \{1 + \sum_t [\pi_{jt}F_{.t}/(V_{ot} + (1+\beta_2)\sum_{k=1}^{J} \pi_{kt} D_{k.})]\}. \tag{3.23}$$

Several analyses have been presented to show the sensitivity of contract prices for a given program to changes in the contractor's business base and/or fixed cost. These analyses are important since other repricing models have ignored these variables. In the first analysis, the price impact of a systematic change in business base is examined. The analysis assumes an expenditure profile of π = (.30,.60,.06,.04), therefore it is expected that altering business base in the second year would have the largest impact on price. Likewise, for the last two years, due to small expenditures, the price is relatively insensitive to the business base changes. The curves, as expected, are downward sloping, implying that program price declines as business base increases (See Figure 3.3). This reflects the fact that in-plant fixed cost is spread over a large number of production units as business base increases.

The same kind of sensitivity analysis is performed for fixed cost, while holding business base constant. As expected, a rise in price with increases in fixed cost is observed. The reason for this increase is that there is more fixed cost to allocate to the same number of units (See Figure 3.4). These analyses demonstrate that annual contract prices are quite sensitive to even moderate changes in both business

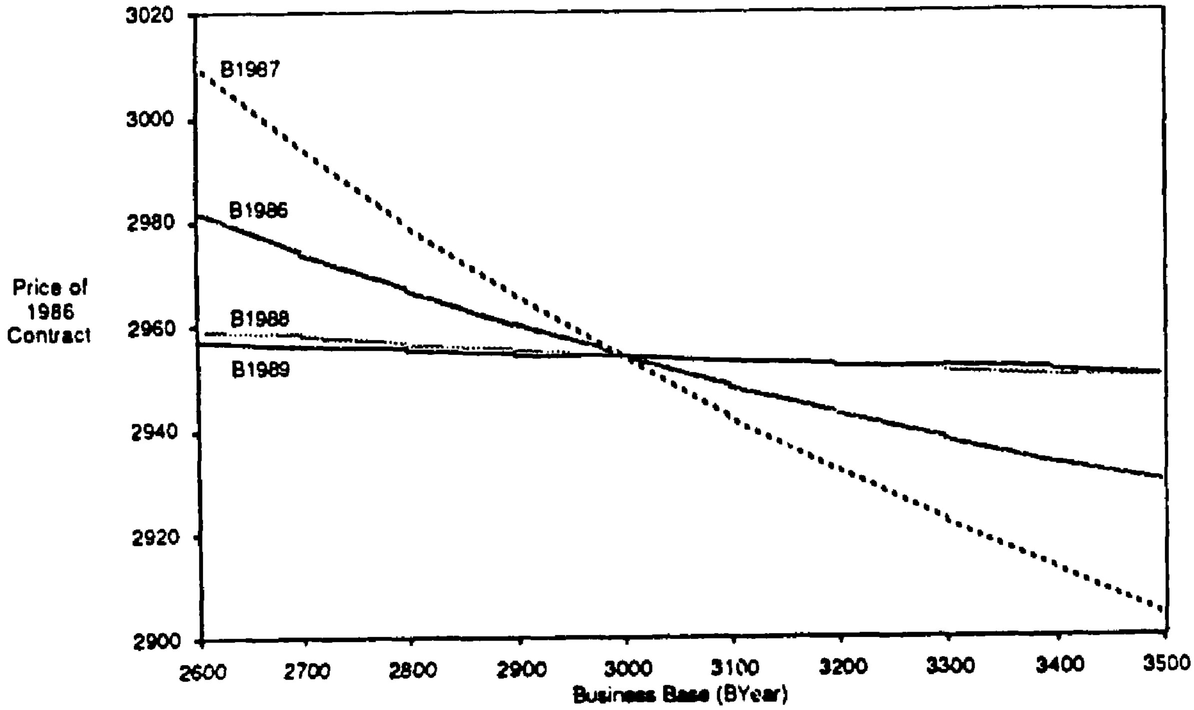


Figure 3.3 Sensitivity of the Contract Prices to Changes in Business Base

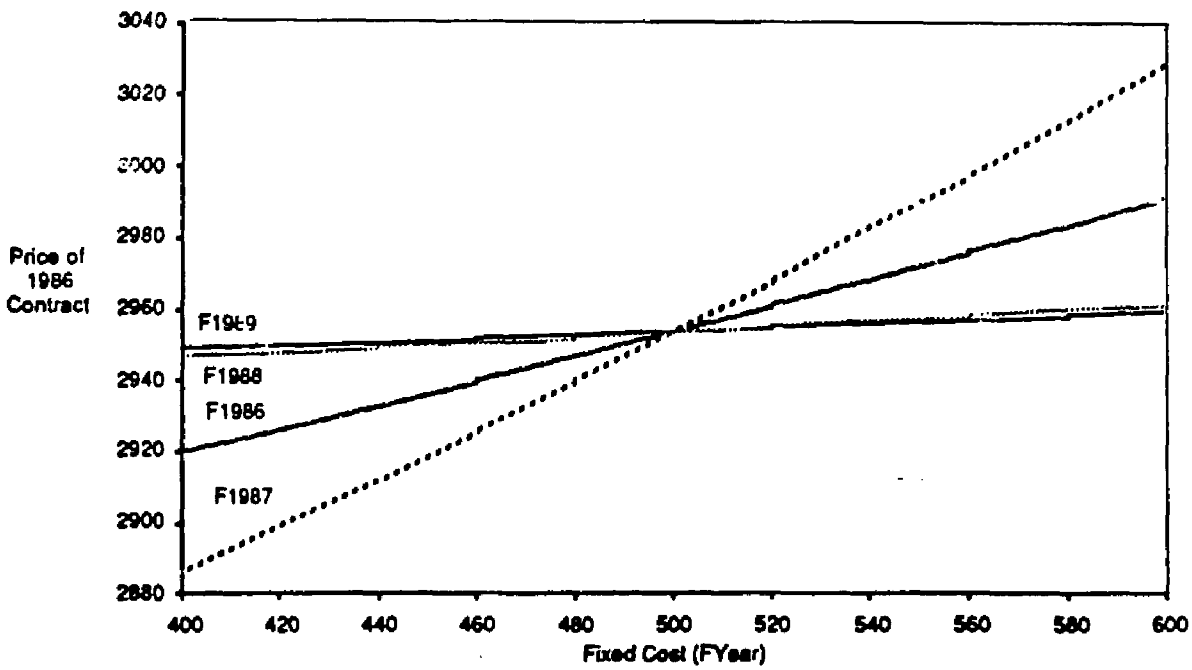


Figure 3.4 Sensitivity of the Contract Prices to Changes in Fixed Cost

base and fixed cost. The data for these analyses are hypothetical. The data base is generated by the Balut, et al. model using default parameter values.

The Balut, et al. model is mathematically simplistic and easy to interpret. The main reason for this simplicity is the assumption of constant production rate. This may be an unrealistic assumption since for most made-to-order programs, particularly aircraft production programs, production rate varies over time and within procurement lot. However, since this model identifies and allocates fixed cost to the contracts, it should be considered as a significant contribution to this research area. The authors illustrate how sensitive the prices are to the changes in business base or fixed cost. These effects have been ignored by more traditional approaches.

This research differs from the regression approach to repricing aircraft in several respects. The regression approach is summarized in the works of Ayres, et al. (1984), Bemis (1981), Bemis (1983), Bohn and Kratz (1984), Bolton (1985), Cox and Gansler (1981), Cox et al. (1981), Gardner (1985), Kratz and Bohn (1985), and Smith (1976). Most aircraft data are by procurement lot. Most convenient ad hoc models require unit data. Since unit data are not available, researchers often try to force the improper model on the data, or they try to alter the data to accommodate the convenient model. Balut et al. avoids these problems by constructing a model that actually generates values for total lot cost. In this respect, the Balut, et al. model avoids the methodological problems of the ad hoc approaches.

Still, the Balut, et al. approach can only be considered to be an approximation since variable costs are modeled with a learning curve. It is well known that the learning curve assumes a constant production rate, therefore the Balut, et al. model assumes constant production rate within each procurement lot. It is also well known that production rate varies over time in aircraft production [Womer, et al., (1986)]. Therefore, the inconsistency is in equation (3.15). This equation must be altered to encompass changes in program production rate. This alteration is the major contribution of this dissertation.

CHAPTER IV

MODEL MOTIVATION AND A THEORETICAL MODEL

The theoretical importance of the inclusion of production rate in cost models is emphasized in the literature review. This importance, with respect to a made-to-order repricing model, is demonstrated below with a numerical example. To show the implications of disregarding the production rate effect, an example is generated and analyzed by using a model that ignores production rate effects.

In this scenario, the government is contemplating procurement of 400 aircraft from a given contractor and receives price estimates as shown in Table 4.1. The plan is to place 100 aircraft under contract each year for the next four years. There are also two other alternatives that are being considered; buying 200 aircraft per year for two years or buying 50 per year for eight years.

TABLE 4.1
Plan and Alternatives

Contract Year	Current Quantity	Planned Dollars	Alternative Quantities Alt 1	Alt 2
1986	100	3080	200	50
1987	100	2300	200	50
1988	100	2060		50
1989	100	1915		50
1990				50
1991				50
1992				50
1993				50

The variable cost by lot and by year and the summary data by lot for the given three cases are calculated by using the Balut, et al. (1986) model. The results are presented in Tables 4.2, 4.3 and 4.4

Estimates derived for the three alternative contracts are given in the above tables. Note that, even though total cost is different for each case, it is only because of the changes in fixed cost. Total variable cost for each alternative remains the same. This is because variable cost is assumed to vary only with cumulative quantity, i.e., the model employs only the learning curve to estimate variable cost. Since 400 units are produced in each scenario, the variable cost is obtained by "moving" 400 units down the learning curve.

This example provides the motivation for the theoretical model that is presented in this chapter. The basic idea is to provide an alternative to the learning curve; an alternative which permits production rate variations throughout the lot production period. A key reference is the work of Womer (1979) as presented in the literature review. The difference, however, is that the Womer model is a continuous time control model while in this application, a discrete time prediction model is needed. This implies a discrete dynamic programming approach such as that advocated by Gulledge, et al. (1985) and Womer, et al. (1986).

A Discrete-Time Production Model With Learning

The basic modeling relationship is assumed to be a type of production function:

TABLE 4.2
Variable Cost by Lot and by Year and Summary Data by Lot
(Current Model)

Annual Lot Size	100	100	100	100
Year	1986	1987	1988	1989
1986	681	0	0	0
1987	1067	510	0	0
1988	444	779	459	0
1989	273	332	720	429
1990	0	204	299	672
1991	0	0	184	280
1992	0	0	0	172
Total Variable Cost	2465	1846	1662	1553
Unit Variable Cost	24.7	18.5	16.6	15.5
Total Fixed Cost	615	451	398	364
Total Cost	3080	2297	2060	1917
Unit Cost	30.8	23.0	20.6	19.2

TABLE 4.3
Variable Cost by Lot and by Year and Summary Data by Lot
(Alternative 1)

Annual Lot Size	200	200
Year	1986	1987
1986	1191	0
1987	1866	888
1988	776	1392
1990	478	579
1991	0	356
Total Variable Cost	4312	3215
Unit Variable Cost	21.6	16.1
Total Fixed Cost	872	749
Total Cost	5184	3965
Unit Cost	25.9	19.8

TABLE 4.4
Variable Cost by Lot and by Year and Summary Data by Lot
(Alternative 2)

Annual Lot Size	50	50	50	50	50	50	50	50
Year	1986	1987	1988	1989	1990	1991	1992	1993
1986	389	0	0	0	0	0	0	0
1987	609	293	0	0	0	0	0	0
1988	253	459	264	0	0	0	0	0
1989	156	191	413	246	0	0	0	0
1990	0	117	172	386	234	0	0	0
1991	0	0	106	161	367	225	0	0
1992	0	0	0	99	153	353	218	0
1993	0	0	0	0	94	147	341	212
1994	0	0	0	0	0	90	142	331
1995	0	0	0	0	0	0	87	138
1996	0	0	0	0	0	0	0	85
Tot. Var. Cost	1406	1059	954	892	848	814	788	765
Unit Var. Cost	28.1	21.2	19.1	17.8	17.0	16.3	15.8	15.3
Tot. Fixed Cost	477	365	303	237	179	151	142	143
Total Cost	1883	1424	1258	1129	1027	966	930	908
Unit Cost	37.7	28.5	25.2	22.6	20.5	19.3	18.6	18.2

$$q_i = F(X_i, T_i, i) \tag{4.1}$$

where q_i = output rate for unit i,

X_i = variable resources required to produce unit i,

T_i = production completion date of the ith unit,

i = the production sequence number of the ith unit.

This production function is assumed to have the usual limiting and continuity properties. In addition, the first and second derivatives are assumed to conform the usual direction of change as presented by Alchian's (1959) propositions (see literature review).

The above production function considers both learning and production rate. In this sense the framework is more detailed than most of the models in the literature. However, it is worth noting that equation (4.1) does not include information on all of the factors that affect cost. There are other factors such as resource prices, subcontracting decisions, etc. that can affect cost on the overall production program. However, for the level of detail required in this modeling effort, it is sufficient to use a production function such as equation (4.1).

This model also assumes that the time horizon is sufficiently short so that resource prices may be assumed to be constant. In that case, discounted cost may be measured in units of the variable resource:

$$C = \sum_{i=1}^{V} \frac{X_i}{(1+r)^{T_{i-1}}} \tag{4.2}$$

where r is the discount rate. The idea is to minimize the discounted cost incurred when producing V units by time T_V. The problem may be stated as:

$$\text{Min } C = \sum_{i=1}^{V} \frac{X_i}{(1+r)^{T_{i-1}}} \tag{4.3}$$

s.t.

$$q_i = F(X_i, T_i, i) \tag{4.4}$$

$$X_i \geq 0$$

$$T_0 = 0 \tag{4.5}$$

$$T_V = T.$$

The last two constraints give the boundary conditions that characterize made-to-order production programs. T is the total time required to produce V units.

Let t_i be the time required to produce unit i, then t_i may also be defined as the inverse of production rate (i.e., $1/q_i = t_i$ for every i). This implies that the resource requirement function associated with equation (4.4) may be stated as:

$$X_i = G(t_i, T_i, i). \tag{4.6}$$

This result is used to eliminate the production function side relation from the optimization problem. That is, equation (4.6) is substituted into equation (4.2), and the revised model is stated as

$$\text{Min} \quad C = \sum_{i=1}^{V} \frac{G(t_i, T_i, i)}{(1+r)^{T_{i-1}}} \tag{4.7}$$

s.t.

$$T_0 = 0, \tag{4.8}$$

$$T_V = T.$$

This problem is a dynamic programming problem that can be solved by considering a sequence of static optimization problems.

The state of the system for unit i is defined by T_i; the decision variable at stage i is the time required to produce unit i, t_i. The stage transformation functions are defined by the additive relationship that exists between the time required to produce the unit and cumulative time (the date of production). Using the boundary conditions given in Equation (4.8), the stage transformation functions are defined as follows:

$$T_0 = 0$$

$$T_1 = T_0 + t_1 \quad \rightarrow T_0 = T_1 - t_1 \tag{4.9}$$

$$T_2 = T_1+t_2 \quad \rightarrow T_1 = T_2-t_2$$

$$\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}$$

$$T_V = T_{V-1}+t_V \rightarrow T_{V-1} = T_V-t_V.$$

This leads to the following restatement of the problem

$$\text{Min } C = \sum_{i=1}^{V} \frac{G(t_i,T_i,i)}{(1+r)^{T_{i-1}}} \tag{4.10}$$

s.t.:

$$\begin{array}{ll} T_1 = T_2-t_2 & t_1 = T_1 \geq 0 \\ T_2 = T_3-t_3 & 0 \leq t_2 \leq T_2 \\ \cdot \quad \cdot \quad \cdot & \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot & \cdot \quad \cdot \quad \cdot \\ T_V = T & 0 \leq t_V \leq T. \end{array} \tag{4.11}$$

This is the form for a dynamic programming problem with the return function

$$R_i(t_i,T_i,i) = \frac{G(t_i,T_i,i)}{(1+r)^{T_{i-1}}}, \tag{4.12}$$

and the stage transformation function

$$T_{i-1} = t_i(T_i,t_i) = T_i-t_i. \tag{4.13}$$

To use the relationship between the return function R_i and the decision variable t_i, the problem may be stated in terms of the recursion equations of dynamic programming:

$$f_1(T_1) = \underset{t_1 = T_1}{\text{Min}} \frac{G(t_1,T_1,1)}{(1+r)^{T_1-t_1}} \tag{4.14}$$

$$f_i(T_i) = \underset{0 \le t_i \le T_i}{\text{Min}} \left[\frac{G(t_i, T_i, i)}{(1+r)^{T_{i-1}}} + f_{i-1}(T_i - t_i) \right] \qquad (4.15)$$

for $i = 1,2,\ldots,V$ with $T_V = T$. This recursive relationship represents a sequence of static optimization problems which, in principal, can be solved for the optimal value of decision variables at every stage. A pictorial representation of this problem is presented in Figure 4.1. The formulation and solution of this model appears to be simple in theory. However, depending on the form of the production function, the solution for equation (4.15) may become very difficult. Based on the experience gained in previous research, the production functions will have unknown parameters, and the functional form is usually nonlinear. This usually means that a "messy" nonlinear programming problem must be solved at every stage.

The specific functional form that is employed by this research is a multiplicative form. The justification for this specific form has its origins in recent work of Muth (1983). The particular a priori specification of the production function that has been selected to be applied to the made-to-order production situation is:

$$q_i = AX_i^{1/\gamma}(i)^{\delta}, \qquad (4.16)$$

or in revised form,

$$t_i = A^{-1}X_i^{\gamma}(i)^{-\delta}. \qquad (4.17)$$

After solving equation (4.17) for X_i, the resource requirement function is

$$X_i = A^{-\gamma}(t_i)^{-\gamma}(i)^{-\delta\gamma} \qquad (4.18)$$

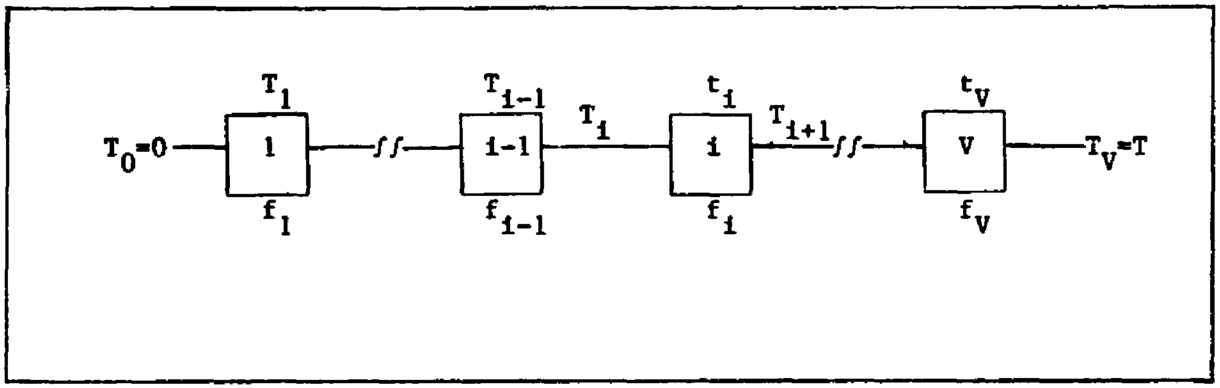


Figure 4.1 Flow Chart of the Dynamic Programming Solution.

where γ is a factor returns parameter, δ is a learning parameter, A is a constant, and i is the production sequence number.

The corresponding dynamic programming problem is:

$$\text{Min } C = \sum_{i=1}^{V} A^{-\gamma}(t_i)^{-\gamma}(i)^{-\delta\gamma}/(1+r)^{T_{i-1}} \tag{4.19}$$

s.t.

$$T_0 = 0, \tag{4.20}$$

$$T_V = T.$$

This model is transformed easily into a dynamic programming formulation as presented in equation (4.10) and (4.11). In this case equation (4.14) becomes

$$f_1(T_1) = \min_{t_1=T_1} A^{-\gamma}t_1^{-\gamma}/(1+r)^{T_1-t_1}, \tag{4.21}$$

and the optimal return function is

$$f_1^*(T_1) = A^{-\gamma}T_1^{-\gamma}.$$

At stage two, the stage transformation function is $T_1=T_2-t_2$ and the optimization problem is

$$F_2(T_2) = \min_{0\le t_2\le T_2} A^{-\gamma}(t_2)^{-\gamma}(2)^{-\delta\gamma} \,/\, (1+r)^{T_2-t_2}+A^{-\gamma}(T_2-t_2)^{-\gamma}. \tag{4.22}$$

However, there is no closed form solution for this problem, and more importantly, the problem turns out to be very difficult to solve numerically. In the special case, however, where r=0, the problem becomes attractive with an easier solution. Theoretically, this assumption seems important however, for practical purposes this model will be developed without discounting. In this case the problem is

$$\text{Min } C = \sum_{i=1}^{V} A^{-\gamma}t_i^{-\gamma}(i)^{-\delta\gamma} \tag{4.23}$$

s.t.

$$\sum_{i=1}^{V} t_i = T_V, \qquad (4.24)$$

$$t_i \geq 0,$$

for $i=1,2,\ldots,V$.

Defining the Lagrangian function as

$$L = \sum_{i=1}^{V} -A^{-\gamma}(t_i)^{-\gamma}(i)^{-\delta\gamma}+\lambda(T_V-\sum_{i=1}^{V} t_i). \qquad (4.25)$$

Following Nemhauser (1966), the first-order conditions are

$$\frac{\partial L}{\partial t_i} = \gamma A^{-\gamma}(t_i)^{-(\gamma+1)}i^{-\delta\gamma}-\lambda = 0 \qquad (4.26)$$

for $i = i-1,\ldots,V$ since t_i will be positive by definition. To check the second-order conditions, the bordered Hessian is evaluated. The $\bar{H}$ matrix is $(V+1) \times (V+1)$ where V is the number of units to be produced.

The form of the Hessian is as follows:

$$|\bar{H}| = \begin{pmatrix} 0 & -1 & \cdots & \cdots & -1 \\ -1 & \frac{\partial^2 L}{\partial t_1^2} & 0 & \cdots & 0 \\ -1 & 0 & \frac{\partial^2 L}{\partial t_2^2} & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ -1 & 0 & \cdots & \cdots & \frac{\partial^2 L}{\partial t_V^2} \end{pmatrix}.$$

By using theorem 8.2.1 in Graybill [(1983) p. 183], the determinant of the above matrix is found as

$$|\bar{H}| = \sum_{j=1}^{V} \prod_{\substack{i=1 \\ i\neq j}}^{V} -\gamma(\gamma+1)A^{-\gamma}t_i^{-(\gamma+2)}i^{-\delta\gamma}. \qquad (4.27)$$

Given the stationary point $(\underline{t}_0, \lambda_0)$ for the Lagrangian function $L(\underline{t}_i, \lambda_0)$ and the bordered Hessian matrix $\bar{H}$ evaluated at $(\underline{t}_0, \lambda_0)$, then $\underline{t}_0$ is a minimum point if, starting with the principal minor determinant of order (2m+1), the last (n-m) principal minor determinants of $\bar{H}$ have the sign of $(-1)^m$ where n is the number of variables and m is the number of constraints. Since m=1 in this model, the last (n-1) principal minor determinants of the $|\bar{H}|$ should be negative for a minimum. Equation (4.27) yields a negative value for every value of γ and δ as long as $\gamma > 0$. Therefore equation (4.26) satisfies the second-order conditions for a minimum.

After solving the first-order conditions [Equation (4.26)], the following solution is obtained for the time required to produce each unit:

$$t_i = [(\lambda/\gamma)\ A^{-\gamma}(i)^{-\delta\gamma}]^{-1/(\gamma+1)} . \tag{4.28}$$

The strategy is to find an expression for λ that is a function of the parameters, and use this expression to eliminate λ from equation (4.28). Using the relationship

$$\sum_{i=1}^{V} t_i = T_V, \tag{4.29}$$

equation (4.28) may be summed on both sides. The resulting expression is

$$\sum_{i=1}^{V} t_i = \sum_{i=1}^{V} [(\lambda/\gamma)A^{-\gamma}(i)^{-\delta\gamma}]^{-1/(\gamma+1)} = T_V. \tag{4.30}$$

Equation (4.30) may be solved for λ as

$$\lambda = T_V^{-\delta-1}[\sum_{i=1}^{V} (\gamma^{-1}A^{\gamma}(i)^{\delta\gamma})^{-1/(\gamma+1)}]^{\gamma+1} . \tag{4.31}$$

This implies

$$t_i = T_V \, (i)^{-\delta\gamma/(\gamma+1)} \; / \; [\sum_{i=1}^{V} (i)^{-\delta\gamma/(\gamma+1)}]. \qquad (4.32)$$

Equation (4.32) is the basic relationship for this model. Together with equation (4.18), they build the basis for the cost estimation procedure. Since $1/t_i$ is the production rate for unit i, the model considers the impact of production rate explicitly. The t_i value in equation (4.32) is the reciprocal of optimum production rate for unit i that minimizes the total cost of the production program. The validity of assuming that the contractor follows the optimum production path has been discussed in earlier sections.

Equations (4.32) and (4.18) are the primary relationships that define the cost model. The integration of production rate (explicitly through the t_i values) with learning should improve the results compared to those models that use only the learning curve. In some cases, average cost may actually increase with volume even in the presence of learning. The reason is that cost decreases due to learning are sometimes offset by cost increases due to decreasing returns. Cases like this would be modeled with more precision by using the model presented above. For a production program where production rate remains somewhat constant over the whole production period, the presented model may not produce significantly different results than the model that employs learning only. However, when production rate varies, which is the common case in many made-to-order production programs, the impact of varying production rate on the predictions is more significant. In general, the conclusion is that the importance of production rate as a cost determinant can not be ignored. This will be demonstrated in Chapter V in the context of an aircraft repricing model.

CHAPTER V

REPRICING WITH VARYING LOT PRODUCTION RATE

In this chapter, a revised method is presented for repricing made-to-order procurement programs. The methodology uses the mathematical model presented in the previous chapter. Equation (3.23), which defines the price of contract j, is the basic model relationship. This equation is extended by intergrating production rate as a cost determinant in the estimation of direct variable cost. While the terms related to direct cost are projected with the model presented in Chapter IV, the other factors (i.e., the expenditure profile and fixed cost) are calculated by the same procedure used by the Balut, et al. model that is discussed in Chapter III.

As described in Chapter III, Balut, et al. define the unit learning curve as

$$X_i = ai^b \tag{5.1}$$

where X_i = the direct cost of unit i,

a = the first unit cost, and

b = the learning slope parameter.

The direct cost associated with lot j, D_j, is expressed as

$$D_j = a \sum_{i=Q_{j-1}+1}^{Q_j} (i)^b \tag{5.2}$$

where Q_j is the total quantity under contract in lots up to and including lot j.

The model developed in this study defines the unit direct cost as

$$X_i = A^{-\gamma} t_i^{-\gamma} (i)^{-\delta\gamma}, \tag{5.3}$$

and the direct cost associated with lot j, D_j, is expressed as

$$D_j = A^{-\gamma} \sum_{i=Q_{j-1}+1}^{Q_j} t_i^{-\gamma} (i)^{-\delta\gamma}. \tag{5.4}$$

Note that in equation (5.2) cost depends only on unit number, where in equation (5.4) production rate as well as the unit number are considered as cost determinants.

Strategy for Implementation and Parameter Estimation

In this section a strategy for parameter estimation in equation (5.3) is discussed. The initial presentation concentrates only on the parameter estimation in the production rate augmented learning curve [equation (5.3)]. The methodology for estimating the other parameters in equation (3.23) is identical to that presented in the Balut, et al. (1986) manuscript.

There are several ways to approach parameter estimation for equation (5.3). However, any method that is selected must also consider the simultaneous relationship,

$$t_i = T_V (i)^{-\delta\gamma/(\gamma+1)} \Big/ \sum_{i=1}^{V} (i)^{-\delta\gamma/(\gamma+1)}. \tag{5.5}$$

The implication is that the parameters must be estimated in equations (5.3) and (5.5).

First, note that the estimation problem cannot be trivialized by assuming that the contractor exactly optimizes production rate. If equation (5.5) is substituted into equation (5.3), t_i is eliminated, but the parameters in the resulting expression are not estimable. The parameters are ill-defined due to model degeneracy [see Bard (1974), section 7-18]. There is no short-cut; data on t_i are required in order to obtain the parameter estimates.

There are several strategies to consider for estimation. In the example presented in this chapter, the parameters are estimated recursively. An extension of this research would require writing code to simultaneously estimate the parameters in equations (5.3) and (5.5). For this research, a two-step recursive procedure is selected. The estimation is carried out by reparameterizing equation (5.5) as

$$t_i = T_V(i)^{-\beta_1} \Big/ \Big[\sum_{i=1}^{V} (i)^{-\beta_1}\Big] \tag{5.6}$$

where $\beta_1 = \delta\gamma/(\gamma+1)$.

An estimate for β_1 can be found by applying nonlinear least squares to equation (5.6). The estimate of β_1 from the first stage, $\hat{\beta}_1$, does not provide a separate estimate of returns to scale parameter, γ. Likewise, it does not provide a separate estimate of the learning parameter, δ. Therefore, equation (5.3) is reparameterized to benefit from the result of the first-stage estimation. The appropriate relationship is

$$X_i = \beta_0(t_i)^{-\gamma}(i)^{-\beta_1(\gamma+1)}. \tag{5.7}$$

Equation (5.7) is absorbed in equation (3.23), and the price equation takes the following form:

$$P_j = \beta_0' \sum_{i=Q_{j-1}+1}^{Q_j} (t_i)^{-\gamma}(i)^{-\beta_1(\gamma+1)} \{1+\sum_t[\pi_{jt}F_{.t}/(V_{ot}+\beta_0' \sum_{k=1}^{J} \pi_{kt}D_{k.})]\} \tag{5.8}$$

where $\beta_0' = (1+\beta_2)\beta_0$. For the repricing analysis, the parameters in equation (5.8) are estimates from lot data using the method similar to that suggested by Gallant (1968).

The procedure is summarized in the following outline.

1. Estimate the parameters in equation (5.5) using nonlinear regression.
2. Use the estimate, $\hat{\beta}_1$, from step one to reparameterize equation (5.3) as equation (5.7).
3. Substitute equation (5.7) into equation (3.23) to obtain equation (5.8).
4. Use Gallant's parameter estimation method on equation (5.8).

Given the final parameter estimates, equation (5.8) may be used to investigate various repricing scenarios. The procedure is applied to data on an airframe program in a later section.

Scope of the Application

It is always difficult to locate non-proprietary data for testing models such as the model presented in this dissertation. Current data is never published, and even though the data are usually not classified, it is proprietary, and hence not available for publication.

To circumvent the data availability problem in this research, a masked data base is used. The variable cost values are derived from estimates of direct labor requirements on an actual airframe program. The data for the business base and fixed cost time series are hypothetical, but they are similar to those realized by one defense contractor. This data base suits the needs of this project. The objective of the research is to determine if price projections are sensitive to production rate variations. This can be accomplished with hypothetical data and sensitivity analysis.

Finally, it is noted that the question of model accuracy is really not an issue in this research. As will be seen later, the model of this dissertation provides projections that are very close to those provided by the Balut, et al. model. The Balut, et al. model has been successfully applied over an extended time horizon to several aircraft programs. The same programs will eventually be analyzed with the new model, but this work, by necessity, must be performed at a secure facility, and the results will not be published.

Data for the Application

The C141 airframe program is selected for the application. The C141 program produced 284 aircraft during the six year period from 1962 to 1968. Only one model of the aircraft was produced. Data for this study are drawn from two sources. Orsini (1970) reports direct man-hours per quarter for each of the twelve lots in the C141 program. He also reports a delivery schedule for the aircraft by month. Orsini attributes these data to the C141 Financial Management Reports maintained by the Air Force Plant Representative Office located at the Lockheed-Georgia facility. The schedule of actual aircraft acceptances by month as reported in the OASD (PA&E) publication Acceptance Rates and Tooling Capacity for Selected Military Aircraft (1974) was used to verify the Orsini delivery data. The 284 units were produced in 12

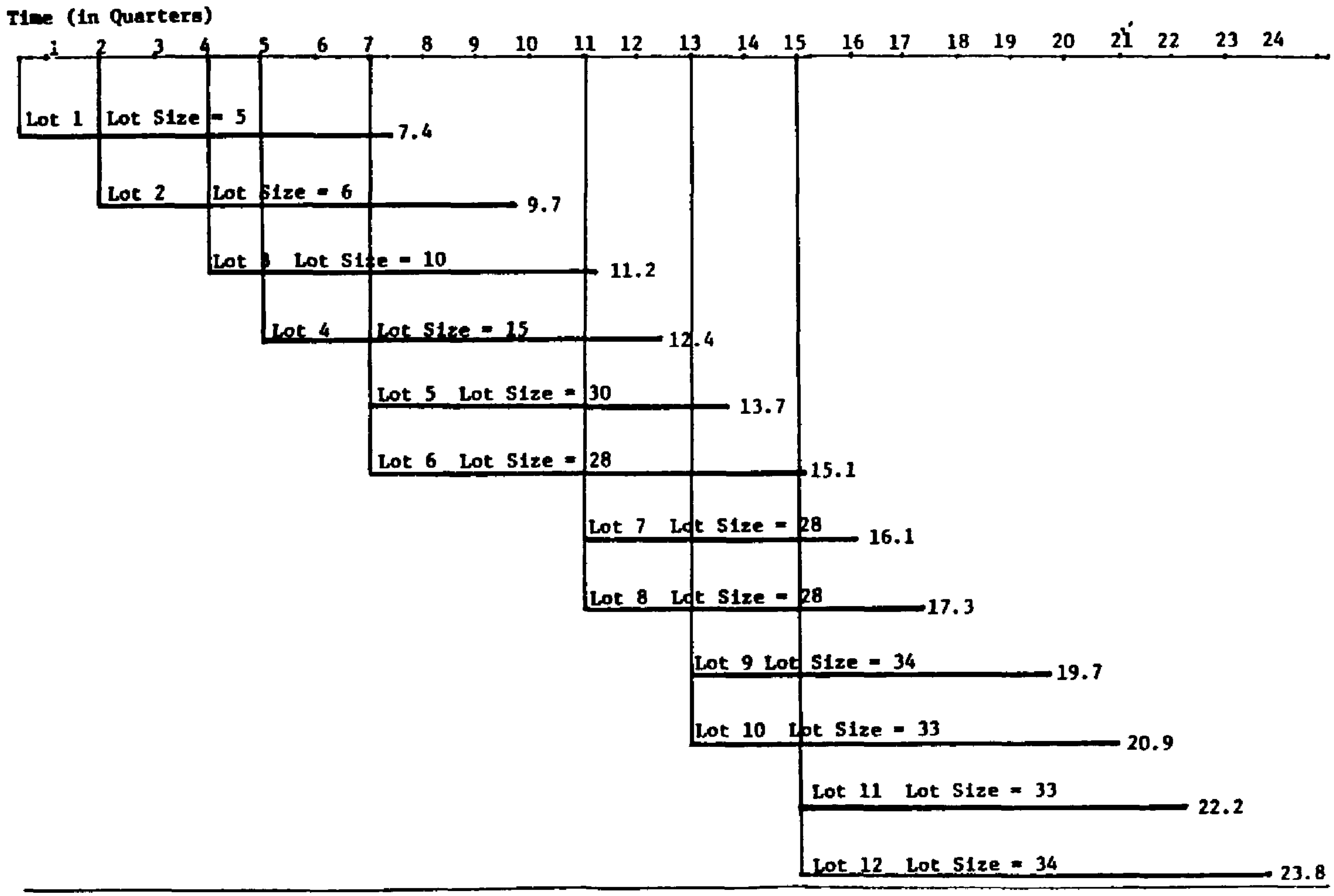


Figure 5.1 Production Lot Time Profiles for the C141 Airframe Program

TABLE 5.1

Delivery Data for the C141 Airframe Program
(In Quarters)

Unit No.	t_i	Unit No.	t_i	Unit No.	t_i
1	0.966666	46	0.191431	91	0.171988
2	0.951675	47	0.191053	92	0.171610
3	0.969999	48	0.190677	93	0.171183
4	0.988345	49	0.190300	94	0.170703
5	0.890013	50	0.189923	95	0.173289
6	0.830554	51	0.189545	96	0.173029
7	0.806639	52	0.189168	97	0.172769
8	0.827169	53	0.188791	98	0.172508
9	0.772692	54	0.188414	99	0.172248
10	0.620989	55	0.187913	100	0.171988
11	0.650769	56	0.187289	101	0.171685
12	0.583330	57	0.186665	102	0.171342
13	0.558051	58	0.186042	103	0.170999
14	0.443892	59	0.185418	104	0.170656
15	0.440833	60	0.184794	105	0.170312
16	0.426663	61	0.184170	106	0.169969
17	0.401384	62	0.183547	107	0.169626
18	0.476105	63	0.182923	108	0.169283
19	0.450833	64	0.182299	109	0.168940
20	0.325557	65	0.181799	110	0.168597
21	0.300281	66	0.181421	111	0.168253
22	0.325001	67	0.181044	112	0.167910
23	0.314075	68	0.180667	113	0.167567
24	0.303148	69	0.180289	114	0.167224
25	0.392222	70	0.179912	115	0.166881
26	0.380105	71	0.179534	116	0.166538
27	0.366797	72	0.179158	117	0.166194
28	0.353490	73	0.178781	118	0.165851
29	0.240183	74	0.178404	119	0.165509
30	0.226875	75	0.178026	120	0.165166
31	0.213569	76	0.177649	121	0.164823
32	0.200261	77	0.177272	122	0.164480
33	0.290927	78	0.176895	123	0.164137
34	0.285559	79	0.176517	124	0.163794
35	0.276482	80	0.176139	125	0.163451
36	0.263704	81	0.175762	126	0.163108
37	0.194349	82	0.175385	127	0.162765
38	0.194108	83	0.175007	128	0.162388
39	0.193867	84	0.174630	129	0.161979
40	0.193627	85	0.174252	130	0.161570
41	0.193318	86	0.173875	131	0.161160
42	0.192940	87	0.173497	132	0.160751
43	0.192563	88	0.173120	133	0.160342
44	0.192186	89	0.172743	134	0.159932
45	0.191808	90	0.172365	135	0.159523

TABLE 5.1 (continued)

Unit No.	t_1	Unit No.	t_1	Unit No.	t_1
136	0.159114	186	0.070466	236	0.081481
137	0.158705	187	0.070134	237	0.081085
138	0.158370	188	0.069802	238	0.080654
139	0.158109	189	0.069470	239	0.080223
140	0.157849	190	0.069138	240	0.079791
141	0.157588	191	0.068805	241	0.079360
142	0.157328	192	0.068474	242	0.078929
143	0.157067	193	0.068142	243	0.078498
144	0.156807	194	0.067810	244	0.078067
145	0.156546	195	0.067478	245	0.077636
146	0.156510	196	0.067146	246	0.077205
147	0.156696	197	0.066814	247	0.076774
148	0.156883	198	0.066482	248	0.076343
149	0.157069	199	0.066150	249	0.075912
150	0.157255	200	0.065818	250	0.075481
151	0.136119	201	0.065486	251	0.075050
152	0.108220	202	0.065154	252	0.074619
153	0.090320	203	0.064821	253	0.074188
154	0.088420	204	0.064489	254	0.073758
155	0.084520	205	0.064157	255	0.073326
156	0.080401	206	0.063825	256	0.072895
157	0.080014	207	0.063493	257	0.072463
158	0.079626	208	0.063161	258	0.072032
159	0.079239	209	0.062829	259	0.071601
160	0.078852	210	0.062496	260	0.071170
161	0.078464	211	0.062164	261	0.070739
162	0.078077	212	0.061832	262	0.070308
163	0.077689	213	0.061500	263	0.069877
164	0.077302	214	0.061168	264	0.069445
165	0.076914	215	0.060836	265	0.069014
166	0.076590	216	0.060504	266	0.068583
167	0.076327	217	0.060171	267	0.068152
168	0.076064	218	0.089275	268	0.067721
169	0.075802	219	0.088817	269	0.067290
170	0.075539	220	0.088331	270	0.066858
171	0.075276	221	0.087845	271	0.066427
172	0.075013	222	0.087358	272	0.065996
173	0.074750	223	0.086872	273	0.065566
174	0.074452	224	0.086386	274	0.065135
175	0.074120	225	0.085899	275	0.064704
176	0.073788	226	0.085413	276	0.064273
177	0.073455	227	0.084927	277	0.063842
178	0.073123	228	0.084441	278	0.063411
179	0.072791	229	0.084016	279	0.062980
180	0.072459	230	0.083654	280	0.062549
181	0.072127	231	0.083292	281	0.062118
182	0.071795	232	0.082930	282	0.062653
183	0.071463	233	0.082568	283	0.064157
184	0.071130	234	0.082205	284	0.066904
185	0.070798	235	0.081843		

production lots. Figure 5.1 illustrates the production period and the spread of these lots over time. The delivery data for each of the units is presented in Table 5.1.

The data on direct man hours spent on each lot were not directly presented in the data base. Orsini (1970) calculated direct man hours for each lot by aggregating quarterly data on each lot. Table 5.2 presents the data on direct man hours (DMH) for each lot, the number of units in each lot, and the direct man hours per unit.

TABLE 5.2

Lot Production Data for the C141 Airframe Program

Lot Number	Direct Man Hours	Lot Size	Average Direct Man Hours
1	211.613	5	42.3225
2	181.241	6	30.2068
3	221.192	10	22.1192
4	250.958	15	16.7306
5	379.754	30	12.6585
6	295.394	28	10.5498
7	247.805	28	8.8502
8	229.634	28	8.2012
9	274.075	34	8.0610
10	255.583	33	7.7449
11	243.539	33	7.3800
12	285.033	30	8.3833

It is evident from the last column that as the number of units produced increases, the DMH/unit decreases. This is consistent with learning curve theory. The only exception is found in lot 12. The reason for the increase in unit labor requirements for lot 12 is what is known as "toe-up". "Toe-up" occurs toward the end of a production program because cost usually increases due to idle workers and equipment, wasted material, and other overhead factors associated with the general de-emphasis of the program.

The data on the 284 actual t_i values are not directly available from the data for the C141 program. Using the delivery dates and lot release dates, the t_i values are approximated from the data. For the starting time of each unit an assumption is made such that every unit in each lot is released in the first half of the lot production period with a uniform release interval. The first unit of each lot is assumed to enter the production process at the lot release date. The delivery dates for each unit (Table 1) are used as the production completion time. Since the production crews work on many units simultaneously within each lot, the difference between the completion and starting dates for each lot are divided by the number of units in that lot to obtain the actual t_i values.

These data provide 284 observations for the first-stage estimation and 12 observations for the second stage. Even though 12 observations for an estimation procedure is small, this limitation is typical of the environment in which the model must be applied. At this point, it is noted that this model is developed to consider ongoing production programs. Even though it is hard to determine exactly how many observations are needed for the estimation, it is obvious that the accuracy of the estimates increases by the number of observations.

Estimation Results

The results given in this section and the sensitivity analyses in a later section were generated by a main-frame based interactive decision support system developed for this research. The program is written in

FORTRAN and performs all of the parameter estimations using Marquardt's (1963) nonlinear estimation algorithm.

In the first stage, the 284 actual t_i values are used as the dependent variable in the estimation of the parameters in equation (5.6). The predicted t_i values are generated by using the $\hat{\beta}_1$ value that minimizes the sum of squared errors (SSE) for equation (5.6). The graph of SSE values as a function of β_1 is given in Figure 5.2. As seen in Figure 2, the SSE function is unimodal and reaches the minimum at $\hat{\beta}_1$ = 0.4859. Having a unimodal SSE function eliminates the risk of inconsistent parameter estimation due to locating different local minimums. The predicted values from equation (5.6) are presented in Figure 5.3. In this figure the smooth broken line represents the predicted t_i values where the more variable darker line represents the actual t_i values. Analyses on the data suggest that the estimation is somewhat sensitive to the accuracy of the t_i values. The degree of this sensitivity has been investigated and the results will be given in the next section.

In the second stage of the estimation process, $\hat{\beta}_1$ from the first step is substituted into equation (5.7). Equation (5.7) is then used in equation (5.8) as previously described. The contour plot of the sum of squared errors (SSE) function for the second stage estimation results is presented in Figure 5.3. Again, a unimodal SSE function has a positive impact on the stability of the model. The values of the estimated parameters and their standard errors for the first and second stages are given in Table 5.3.

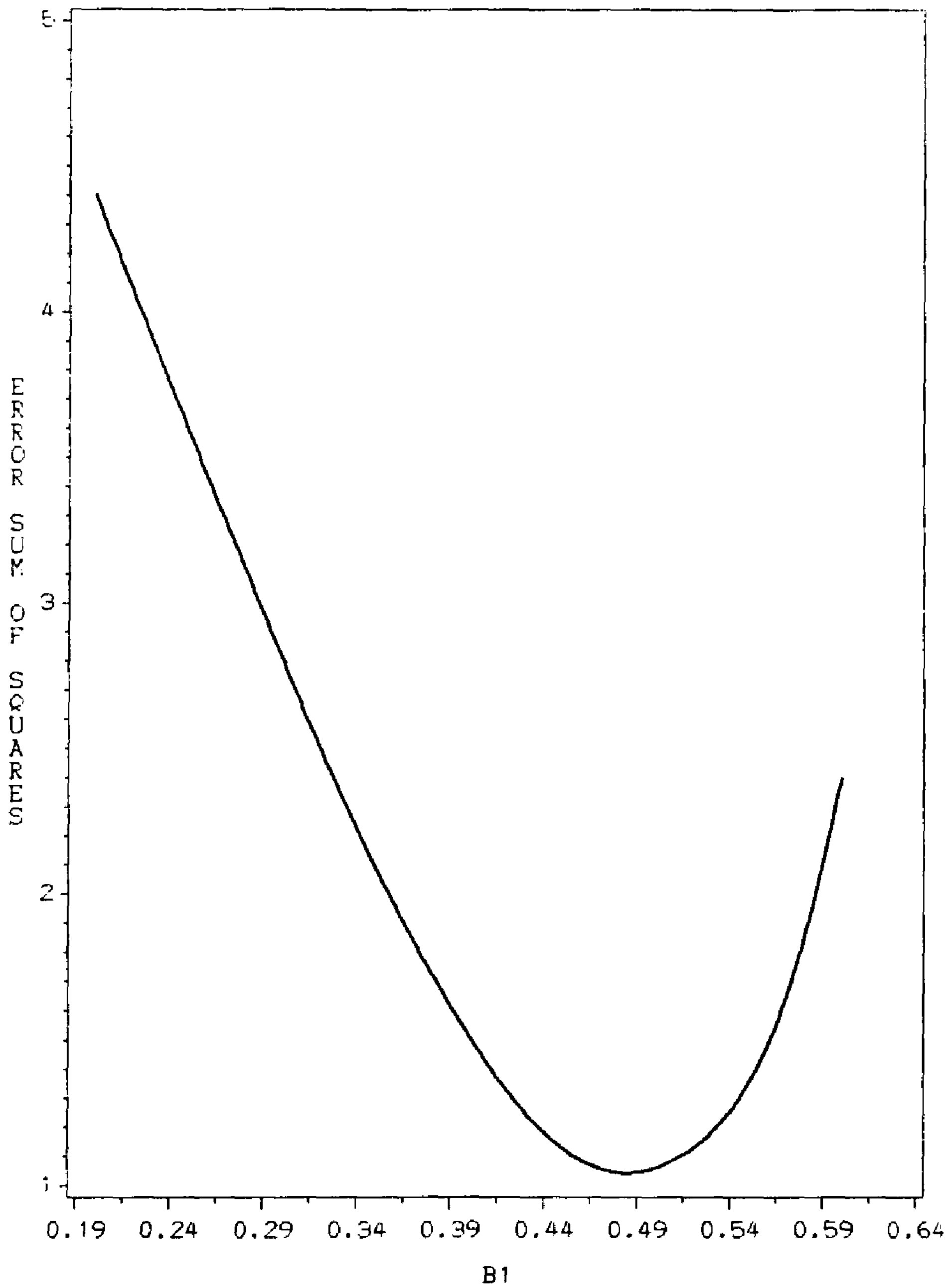


Figure 5.2 Graph of the Error Sum of Squares Function for the Parameter Estimation in Equation (5.6).

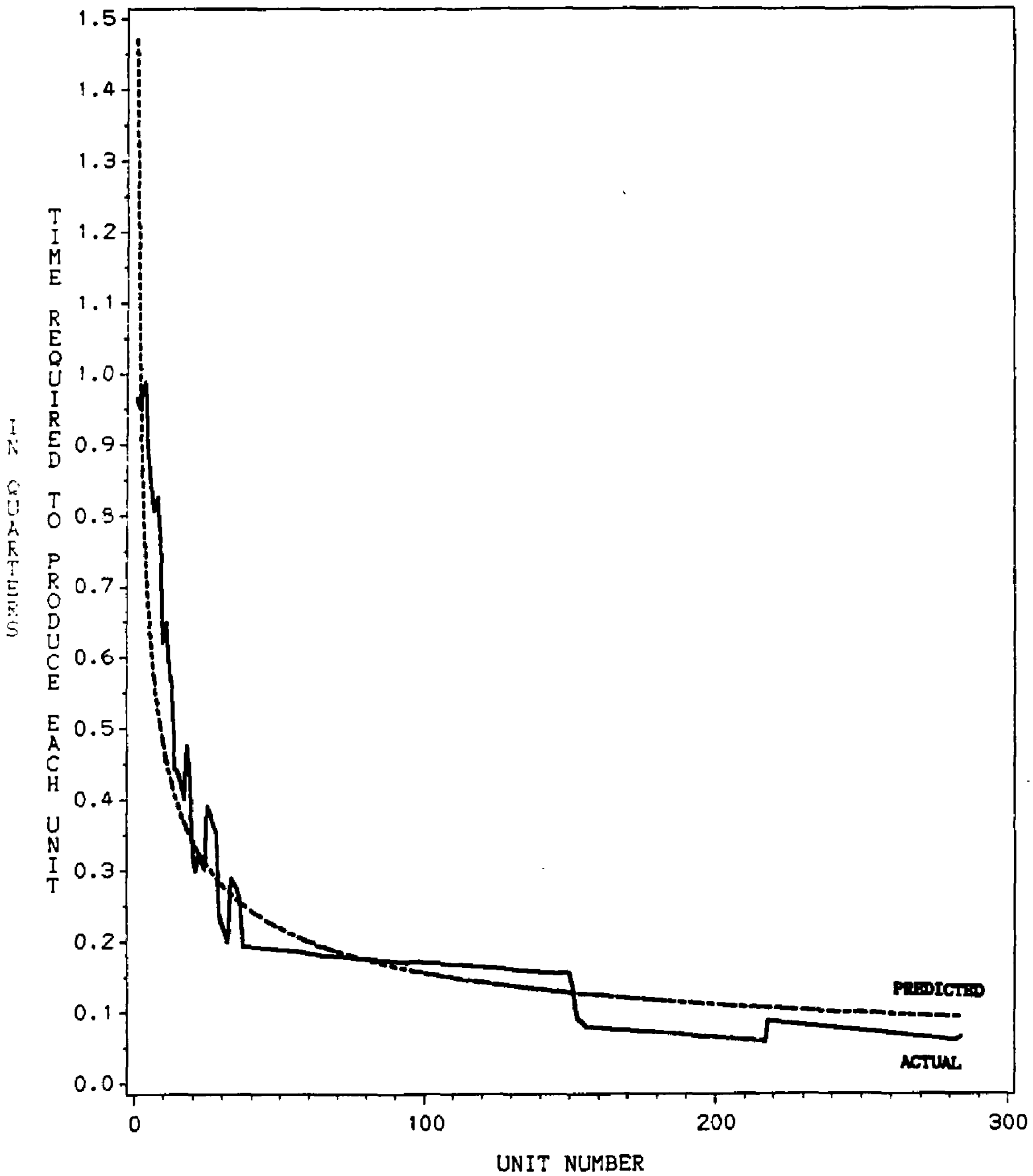
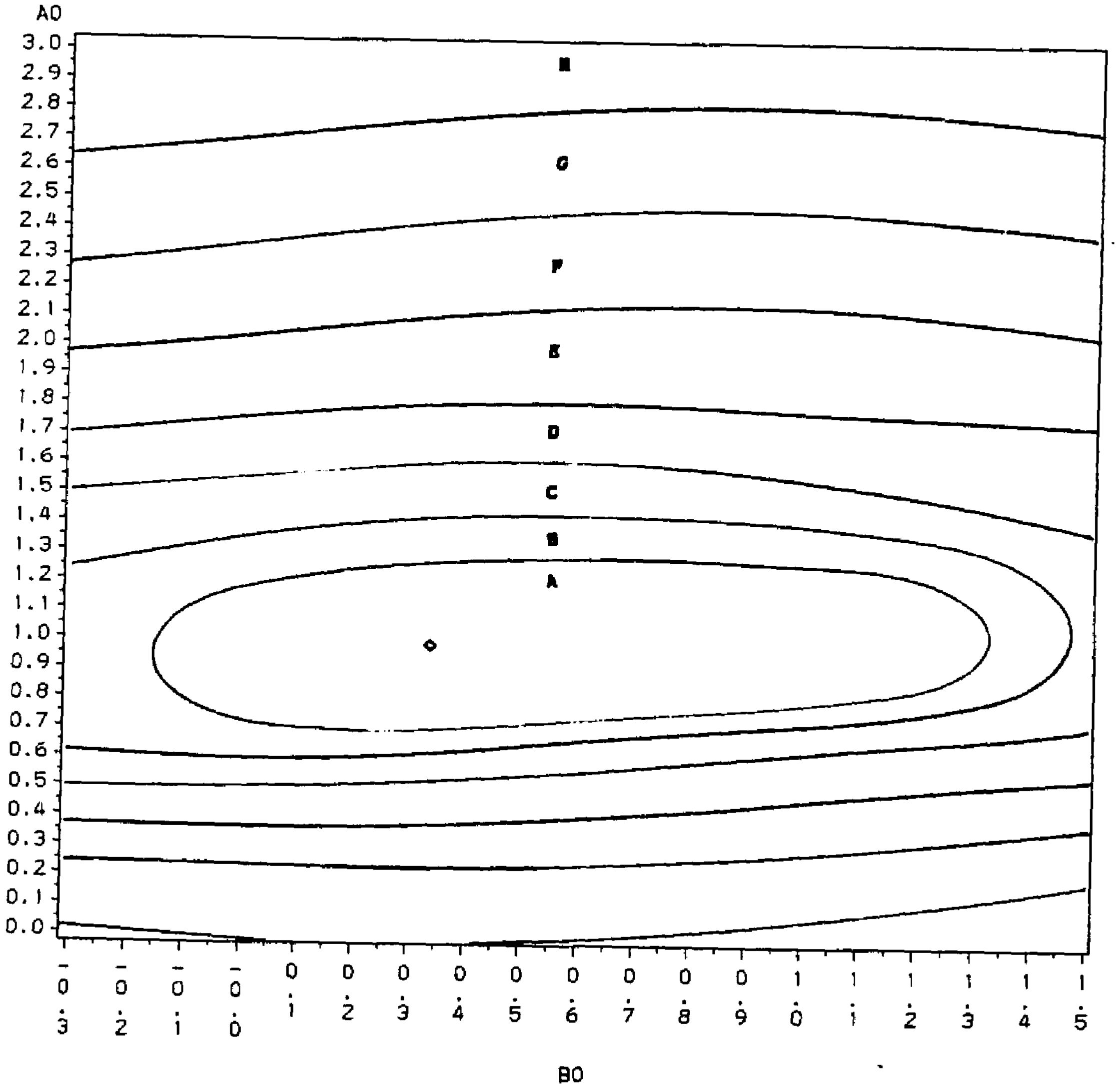


Figure 5.3 Predicted and Actual Values for the Time Required to Produce Each Unit.



SYMBOL	MIN SSE	MAX SSE	SYMBOL	MIN SSE	MAX SSE
A	1.26419	4.39625	E	25.96499	38.64976
B	4.42246	6.60936	F	38.76567	54.33418
C	6.66363	15.71024	G	54.34956	73.46137
D	15.76694	25.63275	H	73.63674	103.20564

Figure 5.4 Contour Plot of the Error Sum of Squares Function for the Parameter Estimation in Equation (5.7).

TABLE 5.3
Estimation Results for the Two-Stage Recursive Estimation

Parameter	Estimate	Standard Error
β_1	0.48591	0.00720
β_0	0.97883	0.05894
γ	0.33716	0.14445

The estimated values from the regression are consistent with a 67% learning curve slope. Both of the estimates in Table 5.3 are significantly different from zero (i.e., production rate and cumulative output). The results are certainly plausible and agree with a priori expectations.

Repricing Results

As a first test, the model is compared with the Balut, et al. (1986) model. A priori it is expected that the models should provide similar predictions over the historical data base. This should be particularly true in the lots where the procurement quantities are stable. That is, if the quantities are stable, there should be little variation in production rate within the lot. The estimated prices for the 12 lots obtained by both models are presented in Table 5.4.

The learning curves associated with these values are not easily plotted because the independent variable at each observation is a sum over all units in the lot. However, as an approximation, the actual and predicted values per-unit are plotted against the estimated lot midpoints in Figure 5.5. At first glance, the results seem similar. However, the model presented in this dissertation tracks the variability

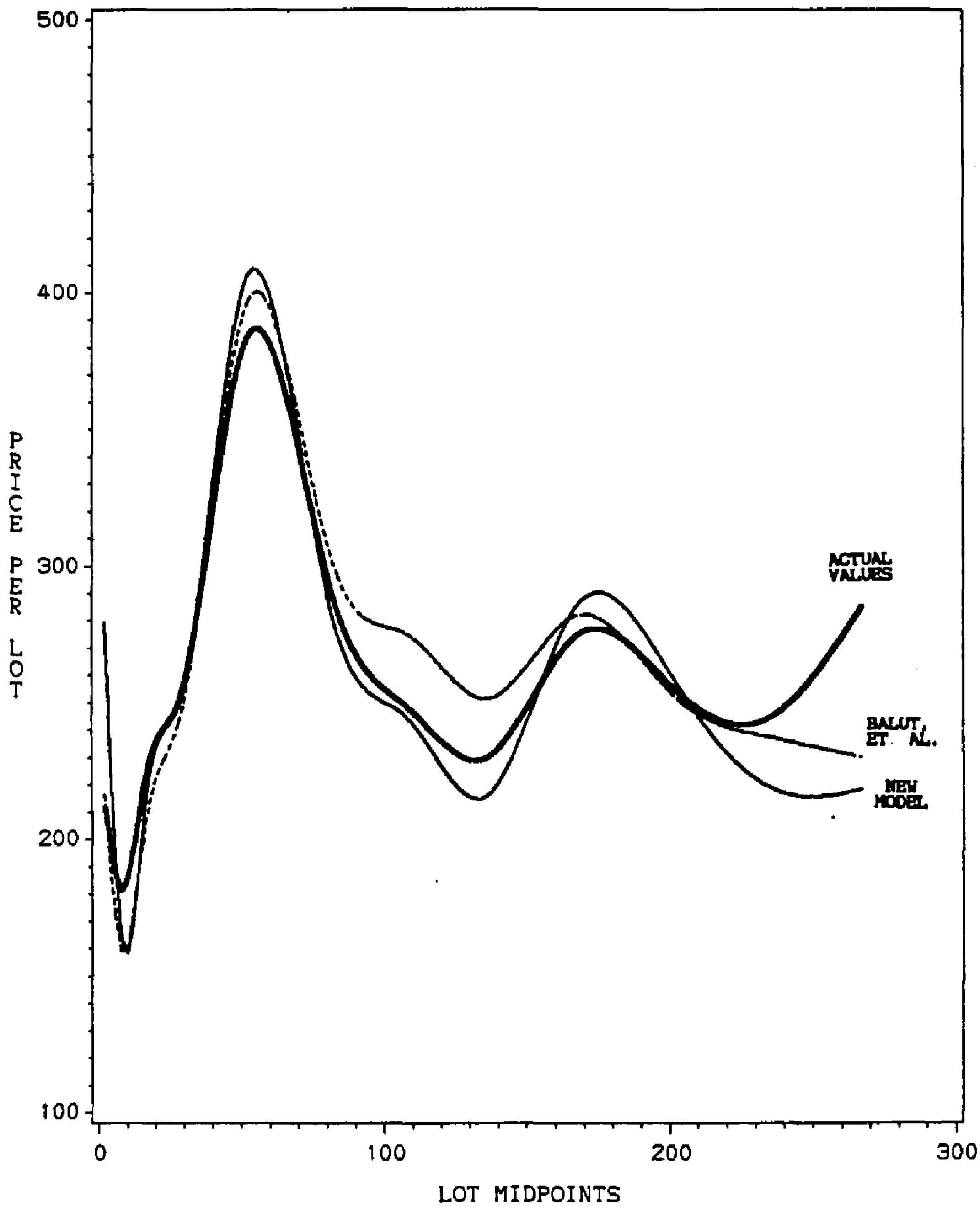


Figure 5.5 Plot of Predicted Lot Price Values by Both Models Versus Actual Data.

TABLE 5.4

Estimated Lot Prices by Both Models

Lot No.	Price Estimates by the Revised Model	Price Estimates by the Balut, et al. Model
1	279.207	216.200
2	162.818	158.700
3	209.344	202.300
4	249.029	242.700
5	401.043	391.200
6	289.745	307.800
7	244.249	274.700
8	215.556	251.200
9	284.511	281.400
10	259.412	253.100
11	219.441	237.300
12	218.062	229.900

in actual values more accurately, particularly in the middle lots. These middle lots were produced during a period when there was much variability in the annual procurement quantities. This tends to support the hypothesis that the new model is more responsive in an environment where procurement quantities are variable.

To further test this hypothesis, an additional sensitivity analysis was performed. All 12 observations from the C141 model were treated as historical data, and four additional lot price projections were computed with each model. Three different cases were run for each model. The annual procurement quantities for the three runs are summarized in Table 5.5.

To price the cases in Table 5.5 using the Balut, et al. model, the variable cost estimates for the last four lots are projected with the learning curve. The projection is more complicated with the model presented in this dissertation. For example, consider Case 2 where the

TABLE 5.5

Hypothetical Annual Procurement Quantities
for Production Lots 14-16

Lot Number	Annual Procurement Quantities Case 1	Case 2	Case 3
13	10	10	10
14	15	20	40
15	10	10	10
16	15	20	40

total number of units to be repriced is 60. Before equation (5.8) can be applied, 60 additional t_i values must be generated. This is accomplished with equation (5.7). The results of the price projections for the three cases are summarized in Tables 5.6 and 5.7.

TABLE 5.6

Price Projections Using the Balut, et al.
Aircraft Repricing Model

Lot Number	Annual Procurement Quantities Case 1	Case 2	Case 3
13	64.9	64.9	64.9
14	95.4	126.8	250.4
15	63.0	62.6	61.1
16	94.1	124.3	240.3

TABLE 5.7

Price Projections Using the Model that Permits
Within Lot Production Rate Variation

Lot Number	Annual Procurement Quantities Case 1	Case 2	Case 3
13	54.74	54.74	55.74
14	79.55	106.13	213.23
15	52.32	52.28	51.70
16	78.18	103.55	201.13

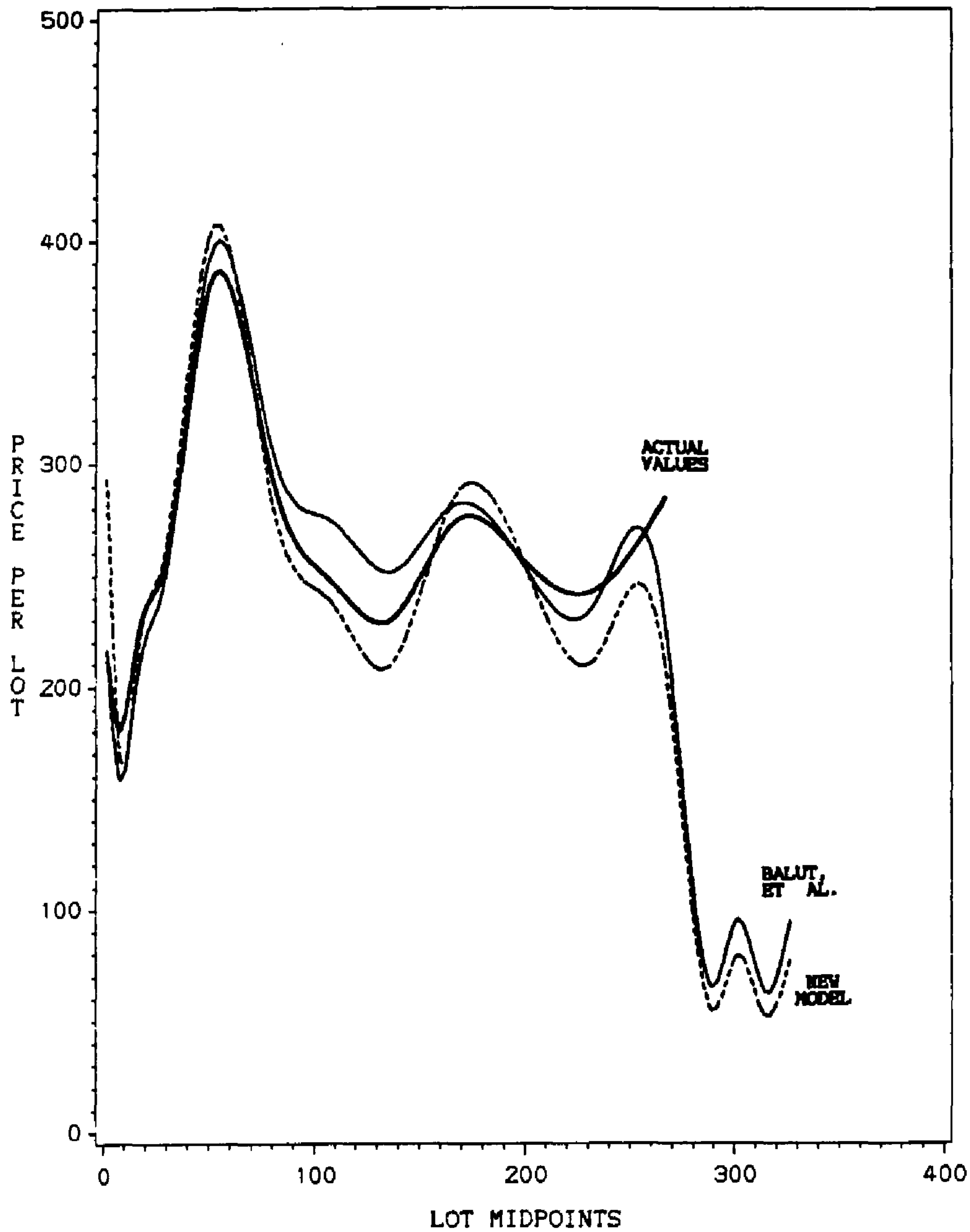


Figure 5.6 **Price Projections for Case 1 Using the Model that Permits Within Lot Production Rate Variation, the Balut, et al. Model, and the Actual Price Values.**

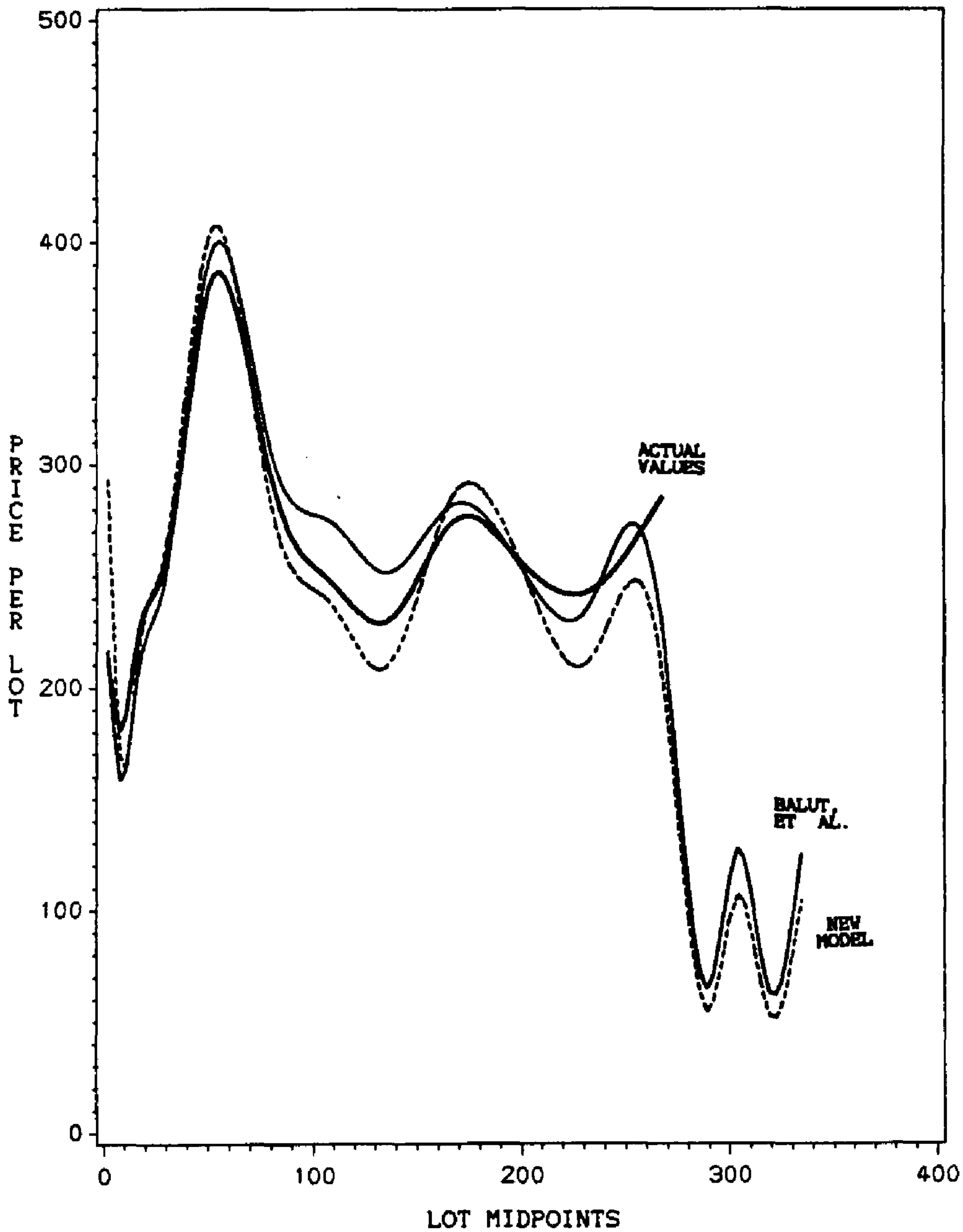


Figure 5.7 Price Projections for Case 2 Using the Model that Permits Within Lot Production Rate Variation, the Balut, et al. Model, and the Actual Price Variation.

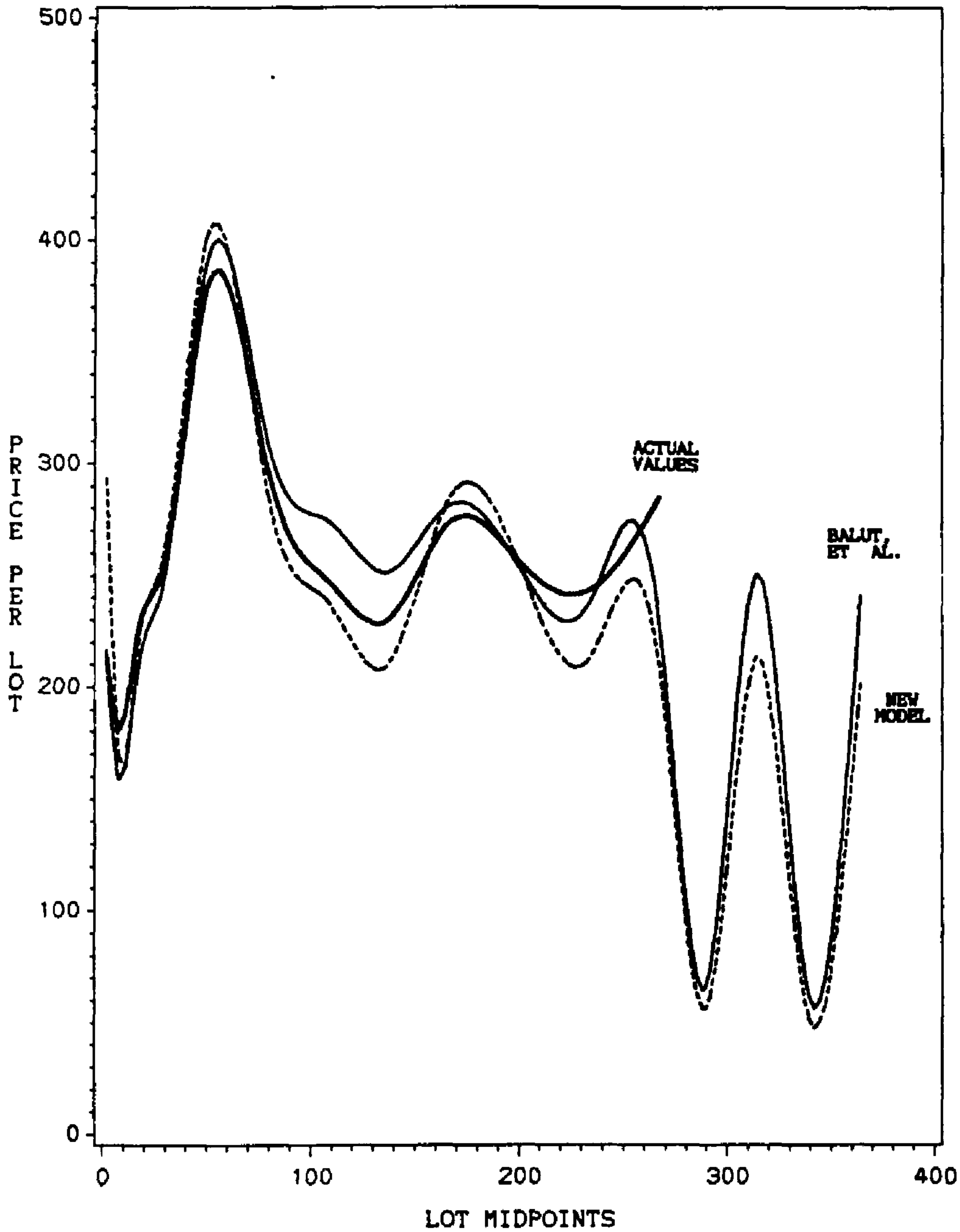


Figure 5.8 Price Projections for Case 3 Using the Model that Permits Within Lot Production Rate Variation, the Balut, et al. Model, and the Actual Price Values.

The implications of these simulations are demonstrated in Figures 5.6 through 5.8. In these graphs, unit annual procurement price is plotted against the lot midpoints of each procurement lot for each of the three cases. Notice that as the variability in the annual procurement quantities increases, the decrease in the relative lot prices predicted by the alternative model developed in this study becomes more significant. The reason for this reduction is the assumption that the contractor follows the optimum production rate to minimize cost.

To further confirm the importance of including a within-lot production rate variable, a Monte Carlo simulation is performed. The importance of production rate is a function of the price sensitivity of alternative t_i values. That is, if price predictions are insensitive to variable t_i values, the implication is that production rate is a relatively unimportant price determinant.

For the simulation, 100 price predictions were generated. All variables within the model were held constant with the exception of the time required to produce each unit, t_i. This analysis also permits confidence interval construction for each of the individual price values.

To obtain the different t_i values to be used as the individual model inputs, normal random error terms were added to the t_i values observed in the data. The generated t_i values are obtained by using the following relationship:

$$t_i' = t_{i\ \text{actual}} + \varepsilon_i \qquad (5.9)$$

where $\varepsilon_i \sim N(0, \sigma_{i\ell}^2)$. $\sigma_{i\ell}^2$ is the variance of the random error for lot

ℓ, $\ell = 1,2,\ldots,12$, which is different for each lot. That is, the variability in the time required to produce each unit would be expected to decrease as more units are produced. ε_i values are generated randomly by using a random number generator presented in Press, et. al. (1986). To estimate the appropriate variance for each lot, the regression analysis in equation (5.7) is repeated using the actual t_i for the dependent variable. The variance to be used in the simulation for each lot is taken to be residual error variance for each lot. The variance for each lot is presented in Table 5.8.

TABLE 5.8

Variances Used in the Construction of t_i' in Equation (5.9)

Lot Number	Variance
1	.32851
2	.05459
3	.06273
4	.05197
5	.01276
6	.00449
7	.00204
8	.00101
9	.01041
10	.00100
11	.00237
12	.00259

As seen in the Table 5.8, the variance terms are generally decreasing as the number of units produced increases. After generating 284 t_i' values for the 100 different simulations, the two-step regression procedure is executed 100 times, and 12 lot prices are computed for each of the simulations. The estimated parameter values for β_1, β_0, and γ for these runs are presented in Table 5.9. The consistency of the signs of the parameters is encouraging in terms of model stability.

Confidence intervals for each price estimate are calculated by appealing to the central limit theorem. These limits are presented in Table 5.10.

The first purpose of this analysis is to determine how sensitive the estimation is to the t_i values. To compare the results, only five of the hundred results that were run are plotted in Figure (5.7). From this figure it is concluded that the t_i values certainly affect the estimation, but the magnitude of this effect is not drastic for small changes in the t_i values. Therefore both conclusions are as expected and desired. The role of t_i as a cost determinant is significant whereas the model shows some flexibility for robustness of the t_i values. It is worth noting that these five results were picked randomly out of one hundred; several other plots like this also were drawn and similar patterns were obtained.

The second reason for this simulation is to observe whether the results from the Balut, et al. model would fall within the confidence intervals of the alternative model. The confidence intervals are obtained by using the central limit theorem and they represent two standard errors. Figure (5.10) graphically illustrates these limits and the actual values, and compares the estimates of the Balut, et al. model. The darker line represents the actual values and the plot symbols represents the estimates of the Balut et al. model.

Except for the first two observations, the results of the presented model is very close to the actual data. The reason for not having good estimates for the first two lots is that the assumption about the t_i values probably does not match exactly with the actual production process. Production of prototype aircraft from the developmental lot

TABLE 5.9

The Parameter Estimates for One Hundred Simulation Runs

β_1	β_0	γ	β_1	β_0	γ
0.46438	73.72931	0.16527	0.48040	80.78752	0.23563
0.46288	73.54172	0.19817	0.50501	91.98950	0.26381
0.51784	104.86227	0.42176	0.53582	112.87532	0.37739
0.44476	66.97636	0.13299	0.47581	78.66634	0.21902
0.48221	80.51509	0.20529	0.46118	72.55978	0.16700
0.46171	72.98198	0.17640	0.44899	68.35283	0.15877
0.47333	76.96582	0.18215	0.54024	124.37807	0.51252
0.52520	111.13809	0.45125	0.48028	81.21492	0.25676
0.53827	118.78563	0.45525	0.46643	75.34508	0.21917
0.52925	109.95552	0.39392	0.46459	74.10297	0.20234
0.48721	84.91446	0.27450	0.47819	79.80240	0.23567
0.49740	88.93182	0.27599	0.45616	71.07724	0.19431
0.45558	70.70731	0.16871	0.48587	84.08804	0.27928
0.46596	75.10385	0.22853	0.46168	72.75078	0.16099
0.53009	104.66904	0.29318	0.42886	62.09525	0.11582
0.49661	91.66246	0.34321	0.50126	91.30678	0.29942
0.52097	104.83115	0.39571	0.51517	101.81770	0.39431
0.45514	69.75514	0.09023	0.51967	107.12730	0.43816
0.48356	82.99779	0.26472	0.47829	80.11383	0.24336
0.45855	71.98323	0.18747	0.49829	91.45076	0.33294
0.47659	79.41431	0.23273	0.49484	90.43051	0.35241
0.46713	75.46095	0.21913	0.48407	81.99754	0.22260
0.48359	80.32956	0.16529	0.43197	62.84959	0.05537
0.50067	93.06961	0.36070	0.48842	86.23883	0.30827
0.48938	85.91319	0.29501	0.46156	72.51106	0.14802
0.44107	65.50258	0.07044	0.45007	68.71608	0.15630
0.51839	106.17606	0.44003	0.50436	91.89757	0.28215
0.48954	86.66530	0.31657	0.50072	82.05070	0.00717
0.46880	76.12851	0.22559	0.49616	90.26575	0.32910
0.54304	122.32925	0.47173	0.49635	90.51231	0.34401
0.49160	86.65352	0.29896	0.49981	91.60828	0.32493
0.48675	83.19751	0.23532	0.48412	83.35368	0.28102
0.47952	80.06198	0.21725	0.47525	78.70743	0.23578
0.47166	76.80695	0.20119	0.47161	77.08421	0.21378
0.46170	73.05963	0.18622	0.47148	77.60704	0.23919
0.45964	72.41592	0.18433	0.52674	111.83038	0.45136
0.50648	97.07613	0.38129	0.42524	61.07327	0.07831
0.50257	91.94258	0.30350	0.46337	74.10866	0.21998
0.48435	83.46138	0.27273	0.46897	76.57785	0.24495
0.47030	76.71104	0.23174	0.53390	113.36145	0.41317
0.52607	101.12312	0.25161	0.50604	96.98645	0.38529
0.48273	83.21347	0.29590	0.48064	80.85170	0.22892
0.51804	101.24034	0.33858	0.53452	116.50867	0.45877
0.50701	97.30510	0.37723	0.44168	65.91112	0.12938
0.48377	82.45593	0.25921	0.48721	83.60156	0.23877
0.41728	58.91260	0.07944	0.48527	83.91388	0.27491
0.47146	77.47063	0.23116	0.51126	96.86151	0.32107
0.48000	80.45290	0.24485	0.52289	105.65495	0.38132
0.49096	85.41219	0.25365	0.51150	98.84743	0.36016
0.50127	90.82219	0.28070	0.52901	110.13922	0.40010

TABLE 5.10

Comparison of Price Estimates of Two Models With the Actual Prices

Lot Number	Confidence Interval Lower Limit	($\mu \pm 2\sigma$) Upper Limit	Estimates of the Balut et al. Model	Actual Prices
1	242.572	331.766	216.19	211.613
2	156.213	177.669	158.61	181.241
3	201.346	224.690	202.32	221.192
4	240.416	263.736	243.61	250.958
5	392.930	406.284	391.25	379.754
6	273.153	303.287	307.77	295.394
7	219.696	266.184	374.72	247.805
8	187.935	240.693	257.25	229.634
9	268.802	290.652	281.41	274.075
10	243.075	263.211	253.14	255.583
11	196.058	234.450	237.31	243.539
12	196.412	229.838	229.92	285.033

may be the reason for this mismatch. Keeping in mind that the Balut, et al. model is considered successful and is widely used in the Department of Defense, improving their results, or even providing comparable estimates lend evidence to the validity of the model developed in this dissertation. Figure (5.5) demonstrates that the revised model provides very accurate estimates, especially in intermediate lots. For these lots, the actual values are within the confidence intervals whereas the Balut, et al. model overestimates the interval. For the last two lots where the lot prices turn upward due to the "toe-up" effect, both models underestimate the actual values.

In this chapter, the results are obtained by using C141 data. The results are very encouraging since the estimates are consistent with the theory. One thing that shouldn't be forgotten is the problem of getting more data to verify the model. The C141 data may not be the best data base to test the production rate effect since it is not known what the

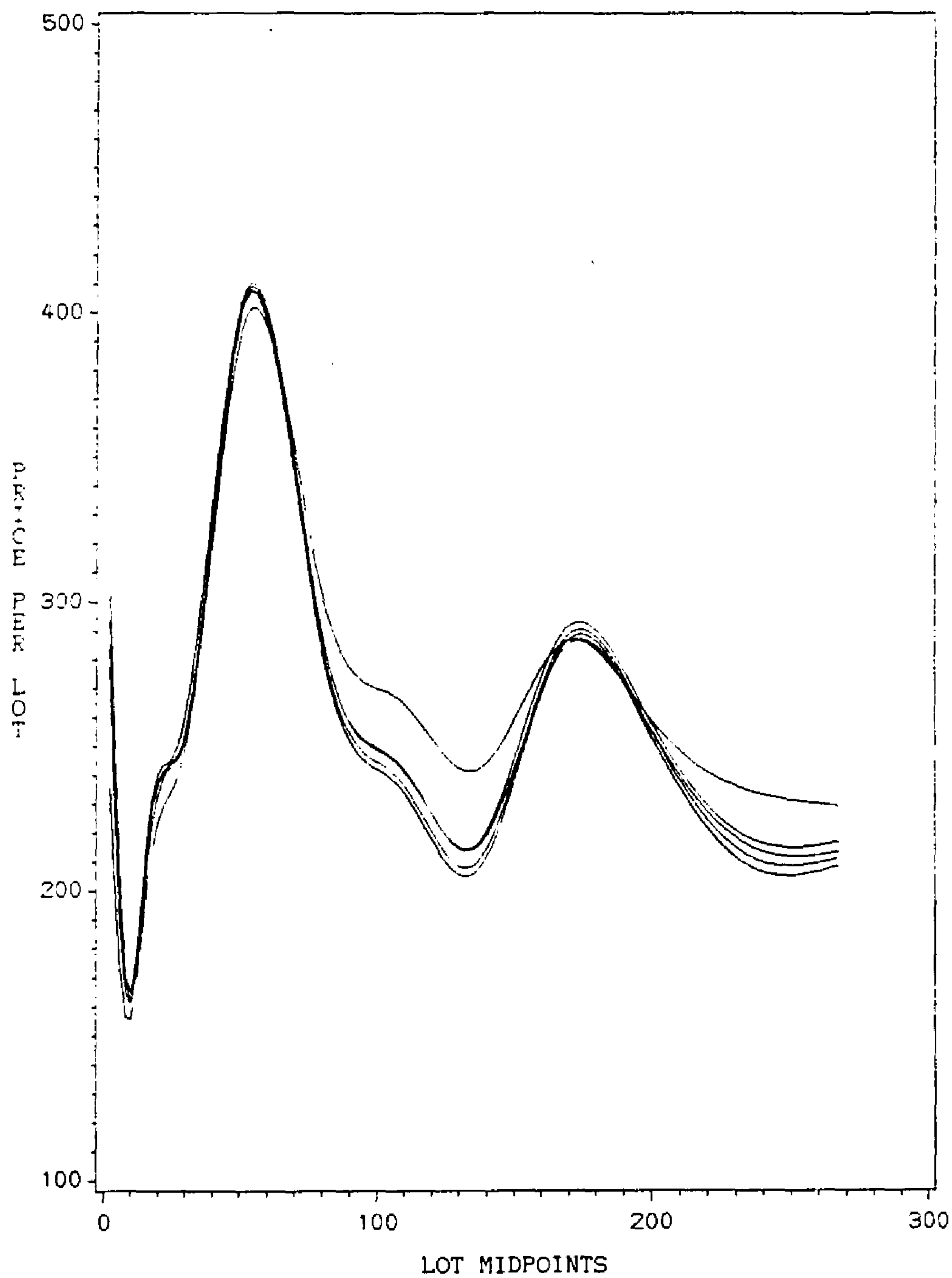


Figure 5.9 Five Simulation Runs Created by Using Equation (5.9).

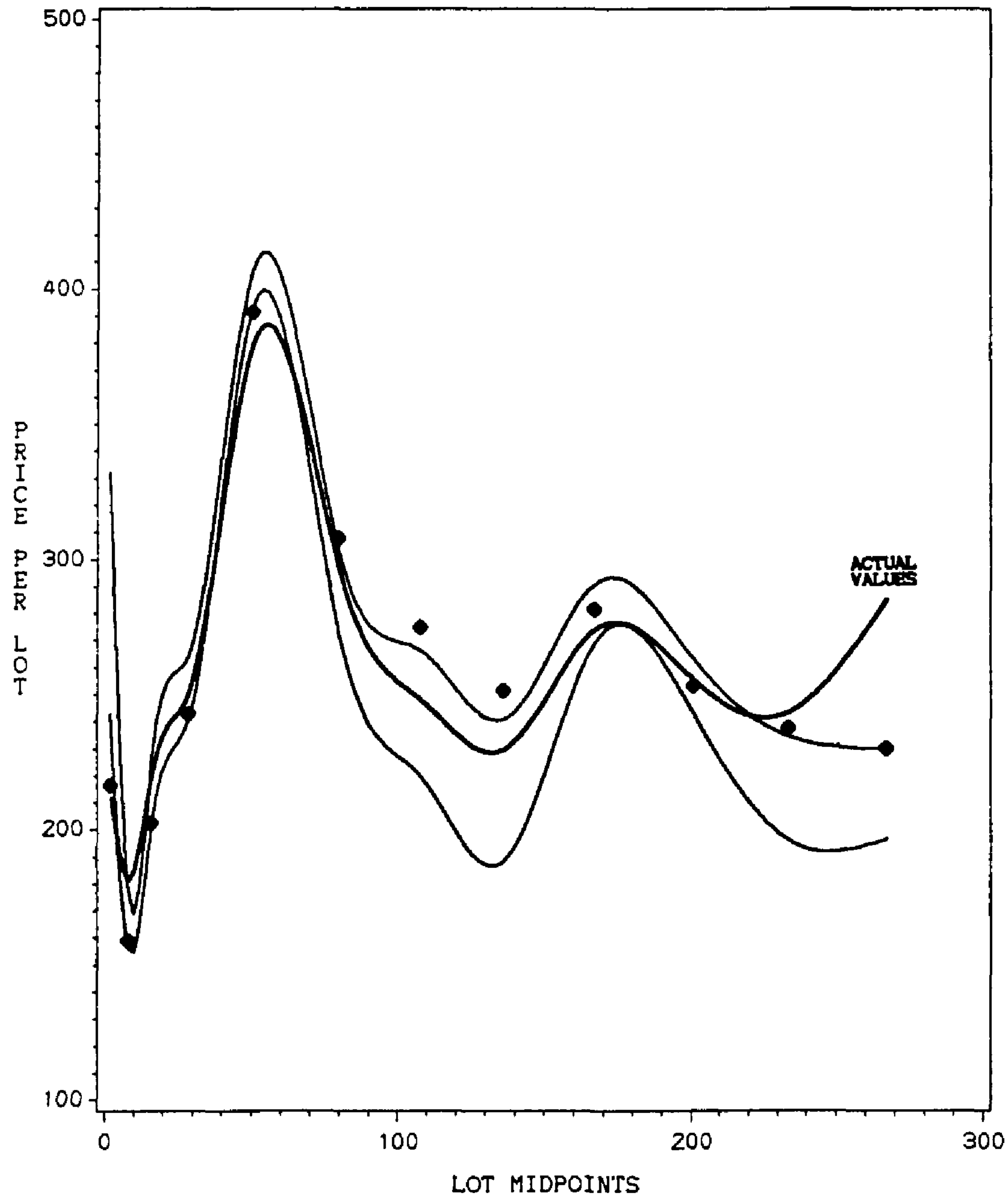


Figure 5.10 Confidence Intervals Created by One Hundred Simulation Runs, Estimates of Balut, et al. Model and Actual Values.

production rate policy was during the production period. Nonetheless, the results are supportive of the theory presented in Chapter IV.

CHAPTER VI

CONCLUSION

The objective of this study was to develop, test, and illustrate the use of a new approach to the repricing of made-to-order production programs. The effort was to extend an existing model, which is currently being used in the Department of Defense, by integrating an important cost determinant, namely production rate, into the model. In Chapter IV, the rationale for the model is provided and the theoretical model is developed. The validity of the estimation results generated by the model is tested in Chapter V. Also, the sensitivity of the model to different production schedules is examined.

The usefulness of this dissertation is apparent. Most made-to-order production programs are subject to alterations and the cost impact of these changes are of concern for cost analysts. Due to the significant differences in accounting systems, it is difficult to examine the cost impact of changes using accounting techniques. A cost model supported by a theoretical framework is useful in tracking the effects of alternative production plans.

The general purpose of the study has been to integrate the production function with a model that projects the variable cost by using a learning hypothesis. The methodology is to minimize the production cost by following an optimal time path of resource use rate. Throughout the modeling effort, it is assumed that the contractor seeks cost minimization. To obtain a closed form solution for the model no discouting assumption is made. Since the model will be used for practical purposes simplicity of the solution is an important factor. For the programs that the relative prices change over the production period this assumption may lead to inaccurate results. The assumption is appropriate for the set of data that has been used in this research. However, the effects of this assumption should be further investigated for other data sets.

The dynamic model is then absorbed in a repricing equation that has been developed by considering other types of concepts such as fixed cost, the expenditure profile, and the in-plant business base. An important aspect of this model is that it can be used to obtain projected costs for different production programs as an alternative to the ongoing program.

An explicit decision support system is developed for the model. A FORTRAN based interactive program is able to provide the analyst with the variable and fixed costs of different production programs based on historical data. This enables the cost analyst, at any stage of the production program, to consider alternative production schedules and their impact on total or unit cost of the product.

By using the decision support system, C141 data is analyzed. Results are as expected and desired. Comparison of these results with the Balut, et al. model and the sensitivity on different delivery schedules illustrated the validity and the reliability of the model.

Future Research

This study, by no means, is the last word on repricing made-to-order production, but it represents one more step in the understanding of the factors that determine cost of production programs. There are several areas that this model might be enhanced.

Theoretically, the assumption of sequential production should be relaxed. This will make the model more representative of the actual production situation. In an actual production, the crew works on more than one unit at a time. Hiring and firing costs may also be included in the model. The impact of hiring and firing cost on total cost, both directly, and through the loss of learning can be examined.

Methodologically, a simultaneous estimation technique may be developed and the results are compared as an alternative to the recursive method employed in this study. Even though it is not clear whether the simultaneous estimation will improve the estimates or not, it certainly is an alternative to be explored.

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VITA

Murat Tarimcilar was born on January 26, 1958 in Bursa, TURKEY. He obtained his Bachelors of Science degree in Industrial Engineering in 1980 from Bogazici University, Istanbul. He worked as a Planning Manager in Karnish Co. from 1980 to 1982. In 1982, he joined Louisiana State University at Baton Rouge. He received a Master of Science in Quantitative Business Analysis from L.S.U. He is now a candidate for the degree of Doctor of Philosophy in the College of Business Administration (QBA) at L.S.U.

DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: M. Murat Tarimcilar

Major Field: Business Administration (Quantitative Business Analysis)

Title of Dissertation: Repricing Made-to-Order Production Programs

Approved:

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

Dan B. Rinks

J R Dorroh

Leslie P. Jones

William W. Williams
for Mine O. Yucel

Date of Examination:

July 10, 1987